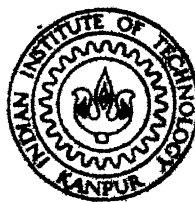


COMPUTER AIDED DESIGN OF WORK DEPENDENT CAMS FOR SINGLE SPINDLE AUTOMATIC MACHINES

by

S. MOHAN KUMAR



DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

APRIL 1987

ME

1987

TH
ME/1987/M

M

K96c

KUM

COM

COMPUTER AIDED DESIGN OF WORK DEPENDENT CAMS FOR SINGLE SPINDLE AUTOMATIC MACHINES

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

by
S. MOHAN KUMAR

to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
APRIL, 1987

To

My Parents

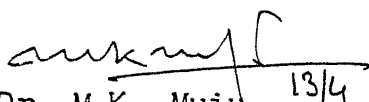
- 2 DEC 1987

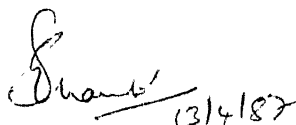
Acc. No. A 88347

ME-1987 -M- KUM- Com

CERTIFICATE

This is to certify that the work entitled "Computer Aided Design of Work Dependent Cams for Single Spindle Automatic Machines" by S. Mohan Kumar has been carried out under our supervision and has not been submitted elsewhere for the award of a degree.


Dr. M.K. Muju 13/4
Professor
Dept. of Mech. Engg.
I.I.T. Kanpur


Dr. S.G. Dhande
Professor
Dept. of Mech. Engg. and
Comp. Sc. and Engg.
I.I.T. Kanpur

ACKNOWLEDGEMENTS

It was Dr. M.K. Muju who rendered me, all kinds of moral support and guidance throughout this work, to whom I am extremely thankful.

I am in large measure indebted to, Dr. S.G. Dhande for putting me on the right path at right times. It was he, who inspired me, at every stage of the work, by rendering constructive criticisms, suggestions and guidance within my comprehension.

I should be grateful to my friend Shastry for his valuable suggestions, while developing the software for the present work. I am immensely grateful to my dearest friends Vinay, Vittal, Madhu, Suresh, Ganesh, Udaya, Silent Madhu, Galagali, Shankar, Shantharam and Ramprasad who were responsible for my happy and memorable stay at I.I.T. Kanpur. I also want to express my thanks to Jayakumar, Poonacha and Appu for their warm-hearted help with their ever-smiling faces during my stay over here.

I am extremely grateful to all of my Kannada Faculty Members, their families and my Kannada friends for making me to feel at home throughout my stay over here, boosting my moral courage and keeping me cool, when I was sick for prolonged period in the burning summer of Kanpur, an exactly a year back.

I am also thankful to the CAD-P Research Engineers, Lekhraj, Smt. Snigdha Biswas, Krishna, Prakash and CAD-P staff Singhji, Bhaskar and Surindar for their all time helping hands and friendly company while developing the package. Finally, I would like to thank Ravi Shukla for his excellent photographing of terminal displays and Srivastavaji for doing an outstanding typing job.

At the end, my thanks are due to the Indian Institute of Technology, Kanpur for providing excellent library facilities and financial assistance, in bringing out this Dissertation work. Also, I wish to thank my parent Institution, Maland College of Engineering, Hassan for its kind extension of study leave.

April, 1987

- Mohan

TABLE OF CONTENTS

	Page
TITLE SHEET	i
CERTIFICATE	ii
ACKNOWLEDGEMENTS	iii
LIST OF TABLES	vi
LIST OF FIGURES	vii
NOMENCLATURE	ix
ABSTRACT	xi
Chapter	
1. INTRODUCTION	1
1.1 Role of Cams in Automatic Machines	1
1.2 Work Dependent and Machine Dependent Inputting Parameters	2
1.2.1 Cam layout sheet and its preparation details	2
1.2.2 Cam modulated linkage for single spindle automatic machines	6
1.3 Statement, Objective and Scope of the Present Work	6
1.4 Organisation of the Present Work	11
2. DESIGN PROCEDURE	13
2.1 Profile Synthesis	13
2.1.1 Determination of contact condition	15
2.1.2 Determination profile coordinates and pressure angle	19
2.2 Curvature Analysis	21
2.3 Cutter Coordinates Determination	30
2.4 Summary About Motion Programs Adopted in the Package	30

	Page
3. SOFTWARE DEVELOPMENT	35
3.1 Implementation Details	35
3.2 Program Structure	35
3.3 Some Important Algorithms	42
3.4 Interactive Features and Getting Used to the Package	46
4. RESULTS AND DISCUSSIONS	52
5. SUMMARY	66
5.1 Conclusions	66
5.2 Scope for Further Work	67
REFERENCES	69
APPENDIX I : AN EXAMPLE OF A TYPICAL CAM LAYOUT SHEET	70
APPENDIX II : MOTION CURVES	72
APPENDIX III: CAM CURVATURE DEFINITION THROUGH DIFFERENTIAL GEOMETRY APPROACH	84

LIST OF TABLES

Number	Title	Page
1.1	Simplified Work Dependent Input Data	9
I	Operation Work Sheet	71

LIST OF FIGURES

Number	Title	Page
1.1	Cam Modulated Linkages	7
1.2	Illustration of Relative Motions in an Automatic Machine	9
2.1	Illustration of System of Coordinate Axes with Symbols	14
2.2	Illustration of Angles and Relative Motion	14
2.3	Schematic Diagram of Rear Slide Cam Modulated Linkage with Symbols	28
2.4	Illustration of Type I, II and III Curves	33
3.1	Flowchart of the Entire Software	36
3.2	Photograph Showing the Display of Turret Cam Modulated Linkage	47
3.3	Photograph Showing the Display of Front Slide Cam Modulated Linkage	47
3.4	Photograph Showing the Display of Rear Slide Cam Modulated Linkage	48
3.5	Photograph Showing the Display of Motion Code Table with Indication of Status at Every Interval	50
3.6	Photograph Showing the Display of a Sample Work Dependent Input Data	50
4.1	Photograph Showing the Display of Interactions While Imposing Restrictions on γ and ρ	54
4.2	Photograph Showing the Display of Turret Cam Profile	54
4.3	Photograph Showing the Zoomed View of Normal Vector, Pressure Angle and Force Vector	56
4.4	Photograph Showing the Display of Turret Cam with Cutter Path	56
4.5	Photograph Showing the Display of Input Motion Curve Plots S , V , A vs. θ_4 for the Turret Cam Design	57
4.6	Photograph Showing the Display of the Plots of γ , ρ , R vs. θ_4 for the Turret Cam Design	57

Number	Title	Page
4.7	Photograph Showing the Zoomed View of Radius of Curvature Variation	59
4.8	Photograph Showing the Display of Work Dependent Input Data for the Front Slide Cam Design	59
4.9	Photograph Showing the Display of Front Slide Cam Profile	60
4.10	Photograph Showing the Input Motion Curve Plots S, V, A vs. θ_4 for the Front Slide Cam Design	60
4.11	Photograph Showing the Display of the Plots of γ, ρ, R vs. θ_4 for the Front Slide Cam Design	62
4.12	Photograph Showing the Display of Work Dependent Input Data of Rear Slide Cam Design	63
4.13	Photograph Showing the Display of Rear Slide Cam Profile	63
4.14	Photograph Showing the Display of Machine Dependent Data for the Turret Cam Design of Example 2	64
4.15	Photograph Showing the Display of Work Dependent Input Data for Turret Cam Design of Example 2	64
4.16	Photograph Showing the Display of Turret Cam Profile of Example 2	65
4.17	Photograph Showing the Display of the Plots of γ, ρ, G_2 vs. θ_4 for the Turret Cam Design of Example 2	65
I	Tool Layout	70
IIa	Modified Constant Velocity (Rise and Fall) Motion Curves	80
IIb	Modified Trapezoidal (Rise and Fall) Motion Curves	81
IIc	Modified Sine (Rise and Fall) Motion Curves	82
IId	Sine (Rise and Fall) Motion Curves	83
III	Surface Representation	86

NOMENCLATURE

f	feed of tool in mms/revolution
v	cutting speed in mms/min
n_s	spindle speed in r.p.m.
N_{tot}	total number of spindle revolutions to produce one component
t_m	main time in seconds
t_{mi}	individual main time in seconds
N_{mi}	individual work spindle speed in revolutions
K_{tot}	total number of divisions on the cam surface
K_i	number of divisions of cam surface necessary for idle strokes
K_{mi}	number of divisions of cam surface necessary for cutting strokes or main time
T_c	total cycle time in seconds to produce one component or for each revolution of the lead cam
S	incremental displacement of the tool slide
V	incremental velocity of the tool slide
A	incremental acceleration of the tool slide
β_4	incremental cam rotation angle
β_2	incremental follower arm angle
d	the follower arm length
β_{20}	the initial arm angle
r_f	the follower roller radius
β	the cam rotation angle at the end of each operation
ω_4	the uniform angular velocity of the cam ($= \dot{\beta}_4$)
ω_{f4}	the uniform angular velocity of the follower
β_2^1	angular velocity of the follower with respect to cam rotation angle ($= d\beta_2/d\beta_4$)

$\ddot{\theta}_2$	angular acceleration of the follower ($= d^2\theta_2/d\theta_4^2$)
θ_2	contact point angle measured from positive X-axis in counter clockwise direction
$\underline{r}^{(P_4)}$	radius vector on the follower surface at the contacting point P
P	the point of contact between cam and the follower roller
\underline{n}	common normal between the cam and the follower surface
$\underline{v}^{(42)}$	the relative velocity vector between cam surface and the follower surface at the point of contact
$R_4^{(P)}$	the equation of the cam surface
X, Y	the cam profile coordinates
ψ	the pressure angle at the point of contact
\underline{e}_4	unit normal on the cam surface at the point of contact
ρ	the radius of curvature of the cam
R	the profile radius of the cam.
r_c	the cutter radius
X_C, Y_C	the cutter coordinates

ABSTRACT

In the present work, a design methodology for designing work dependent cams for single spindle automatic machines has been developed. An interactive graphic software has been developed and implemented, with the programs written in ANSI Fortran 77 language.

The package essentially takes the input in the form of machine dependent data and the work dependent data. The machine dependent data refers to the type of the cam modulated linkage used for driving the tool slide and the work dependent data depends basically upon the geometric descriptions of the workpiece and the type of operations performed on the same. The package incorporates the motion curves as per the universal system of cam calculations. The method adopted for getting the actual motion parameters considerably saves the computation and the comparison times. During feeding of the data, the package provides all possible user-friendly interactions. The important displays like, the cam profile, the input motion curve plots, the output parameter variations etc. are neatly presented to the user. The package accepts restrictions on pressure angle and radius of curvature values and thereby, designs the cams to the necessity of the User. Optional displays, pertaining to the cutter path, for a specified cutter radius but pressure angle, normal vector and force vector on any point of the cam profile can also be obtained using the package. The

display of the main cam profile is strictly as per the conventions followed in the industries.

The cam profile synthesis and radius of curvature evaluations are made using screw theoretic approach and differential geometry respectively. The results thus obtained, are highly accurate from the manufacturing viewpoints also.

Results are presented for typical workpieces produced on single spindle automats and are analysed. In the presently developed work, the scope for further research has also been suggested along with the conclusions.

The software details and other necessary informations about the package are discussed and presented in a neat form. The package has partial self documentation features like meaningful symbols and components at suitable points. In all, the graphical displays, meaningful colour matching is done. For graphic displays, the Tektronix 4109 and 4107 colour graphics terminals are used and the package has been implemented on Norsk Data (Norway)'s a 32-bit Super-Mini Computer.

CHAPTER 1

INTRODUCTION

1.1 Role of Cams in Automatic Machines

In an Automatic machine all working and handling operations are performed in a definite sequence by the mechanical control system, which comprise of cams, gears, linkages and other transmission elements. For all machine operations the major requirements are precise positioning, along with fairly good phase synchronization. To meet these complicated requirements and to offer flexibility in the selection of working periods, dwells and in getting a specific type of output motion cams are the excellent choices. The plate cams are the main controlling elements in bringing up a definite workpiece in all single spindle Automats.

Plate cams are of one piece design unlike drum cams, and are used comparatively for short travel. In single spindle automats the tools are stored in a turret and in a number of cross slides and are put to operation sequentially. The precision of the workpiece produced on any cam controlled automatic is largely dependant upon the accuracy with which the cam profile has been designed and cut. For designing automat cams, the essential inputs will be in the form of the type of component to be manufactured, the kind of tools to be

used, and the type of cam modulated linkage. The former two constitute basically the work dependent input data and the last one the machine dependent or the work independent input data for designing plate cams for all single spindle automatics.

1.2 Work Dependent and Machine Dependent Inputting Parameters

The precision and accuracy of the work produced on a cam controlled automat depends to a great extent upon how intelligently the cam layout sheet has been worked out. A cam layout work sheet essentially produces the necessary work dependent input to design the cams of all automatic machines. Hence a brief presentation of the nature and preparation details of a cam layout sheet follows.

1.2.1 Cam layout sheet and its preparation details

A cam layout worksheet consists of a tool layout and an operation worksheet. A tool layout contains sketches of the work in the various stages of machining operation by operation elements. The order of the individual operations is chosen in such a manner that the component is fully machined using the available turret stations and cross slides. In order to minimise the cycle time several tools have to be used with multi-tool holders, wherever possible. Also the overlapping operations have to be thought off, for not only minimising idle time but also for better finishing operations. Sometimes it may become essential to add to the rigidity of

the workpiece while performing transverse cutting operation using a cross slide, e.g., while thread rolling using a cross slide, performing a drilling operation simultaneously, will add to the rigidity of the workpiece. In some situations, in order to counterbalance cutting forces overlapping of the operations are done.

After thorough considerations like the above ones, the tool layout will be drawn with the approximate extreme positions of the slides. Appendix I shows a typical tooling layout used for manufacturing of a brass valve seat on an automatic screw machine. In a tool layout the order of the working operations and idle actions like feed bar stop will be indicated clearly.

Then follows the operations to be performed from the turret and cross slides which are entered in proper order on a prepared calculation form. This form is called an operation sheet. An operation sheet shows the sketch of the furnished workpiece, indicates the work material and lists all the operation elements in proper sequence in accordance with the tooling layout. It contains properly selected cutting speeds and feeds for each tool as per the specifications¹ of an autostat. The choice of feed² (f) and cutting speeds³ (v)

¹ as per manufacturers.

² Feed is the transverse movement of the tool in mm per revolution of the work spindle.

³ Cutting speed refers to the maximum surface speed of the stock in mm/minute.

mainly depends upon the type of work material used, the amount of surface finish required, the depth of cut to be given, the kind of tool to be used and the amount of rigidity the work has. The spindle speeds found in operation sheet are calculated from the recommended cutting speeds (v) for the appropriate work material and the largest diameter (D) which is to be turned using the simple expression

$$n_g = v/\pi d \text{ r.p.m.}$$

This calculated spindle speed (n_g) is compared with the nearest values of the available speeds and then the selection of actual revolutions available with the regular change gears that come nearest to the estimated number is made.

The travel distances are tabulated in the operation sheet as per the component drawing with suitable allowances depending upon the size of the automat, approach length of the tool and magnitude of the slide travel.

The next important parameter in operation sheet deals, is about work spindle revolutions. Every rotation of the cam results in a workpiece. And because this revolution is governed by the work spindle, the number of spindle revolutions for each operation is determined, to enable the appropriate portion of the cam profile to be calculated. The total number of spindle revolutions (N_{tot}) required for producing one component is governed by the main or non-idle-time or working

time (t_m). This main time is the summation of individual operations times (t_{mi}) which in turn is directly proportional to the individual work spindle speed (N_{mi}) and is calculated using travel strokes of each individual operation (l_i) and the corresponding feed rates (f_i).

Thus the total main time spindle revolutions (N_m) becomes

$$N_m = \sum N_{mi} = \sum (l_i / f_i)$$

Next description in the operation sheet will be pertaining to the distribution of cam surface. It is a usual practice to quote, the angular divisions of the cam surface in terms of 100th divisions or 360th divisions. Denoting ' K_m ' as the number of hundredths required for main time operations and K_i as that of idle time operations. Usually the setting up instructions of any Automat indicate the idle motion time (K_i) against various values of total spindle revolutions. With this, the hundredths required for each main time can be easily calculated using

$$K_{mi} = (K_{tot} - K_i) * N_{mi} / N_m$$

where $K_{tot} = 100$.

The final field in the operation sheet is the total cycle time T_c required to produce one component or for each revolution of cam and is given by

$$T_c = N_{tot} / N_S$$

where $N_{tot} = \frac{N_m * K_{tot}}{K_m}$

For better visualization of the operation sheet Appendix I has to be referred.

1.2.2 Cam modulated linkages for single spindle automats

These are essentially the machine dependant input for designing the cams. The most commonly used follower cam mechanisms are as shown in Figure 1.1a. Figure 1.1a is a typical cam modulated linkage used for turret and front cross slides, whereas Figure 1.1b is one such used for rear cross slides. In some cases in place of gear pairs rotating cam pairs are used. However, in the present work for the purpose of analysis and design the most common gear pairs have been considered.

1.3 Statement, Objective and Scope of the Present Work

An Automat is a programmed machine tool. The programme consists of actuation of a set of tools through programmed motion on a cyclic basis. The total cycle time (T_c) is the sum of non-idle time or main time (t_m) and the idle time (t_i) for each component. The motion could be angular (work-piece rotation or turret head indexing motion) or could be linear along axial (longitudinal movement of turret station), and in transverse (cross slide movement) directions. Figure 1.2 gives the schematic representation of these relative motion of tools and workpiece. Denoting the motion parameters:

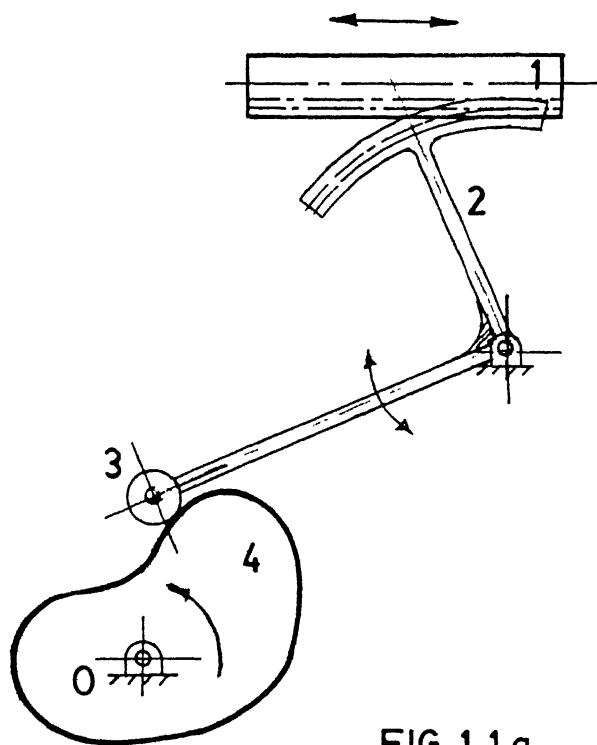


FIG. 1.1 a

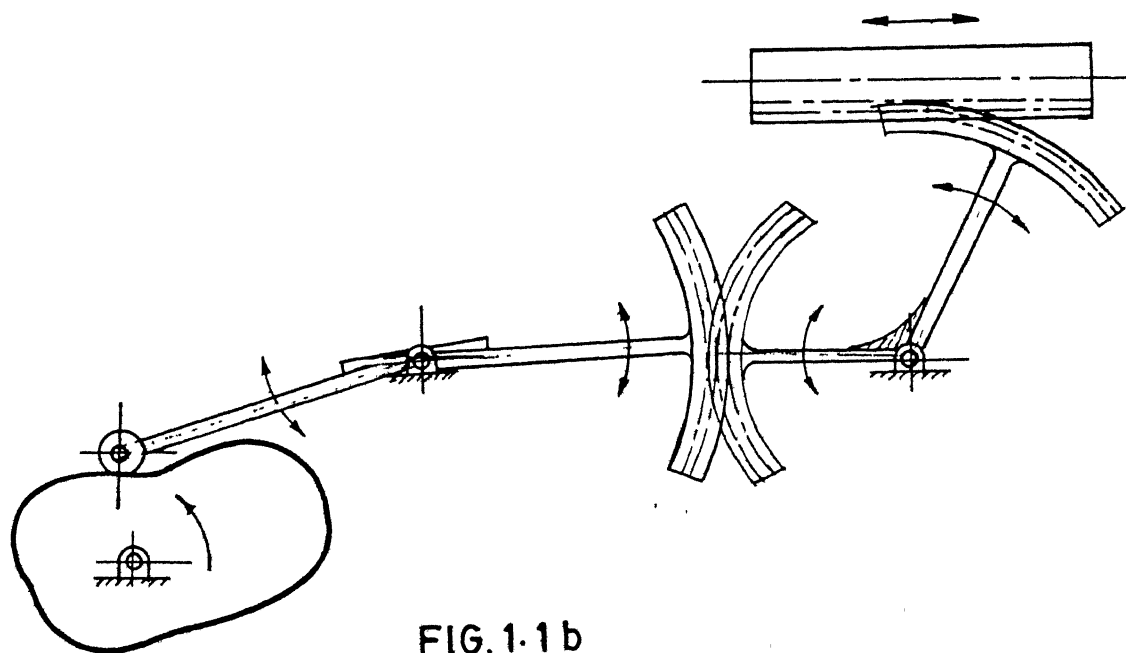


FIG. 1.1 b

FIG. 1.1 CAM MODULATED LINKAGES

by $S_1, S_2, \dots, S_1, \dots, S_N$, where N indicates the number of motions considered in the machine per component output. A motion program describe a curve S_1 Vs. t , as well as $\frac{dS_1}{dt}$ Vs. t and (d^2S_1/dt^2) Vs. t . As there are definite inputting flag positions or operations or spans as described in cam layout sheet for each component, the future task is to assign a suitable motion program to each of these spans. When the cam layout sheet is overviewed in this perspective, the outlook will be as shown in Table 1.1.

Hence, given the cam sheet layout which is essentially work dependent data along with the connected cam modulated linkage, which is work independent, the objective is to devise an interactive software for designing plate cams needed for single spindle automatic machines. The software developed in the present work, designs plate cams to a very high degree of accuracy and is user friendly. The concept of screw theoretic approach [1] has been adopted for the profile synthesis. The package entangles basically the motion curves of three groups of acceleration diagrams of modified trapezoidal, modified sinusoidal and sinusoidal nature having informations of types I, II and III¹ for rise or fall as per universal system²

¹Type I is the stroke which starts from a Dwell and ends in a Dwell;

Type II is the stroke without dwell; and

Type III is a combination curve of I and II.

²The name universal system of cam calculations is to reflect the fact that the first development of the method was done at Universal Match Corporation, U.S.A., specifically to meet the requirements of universal application [2].

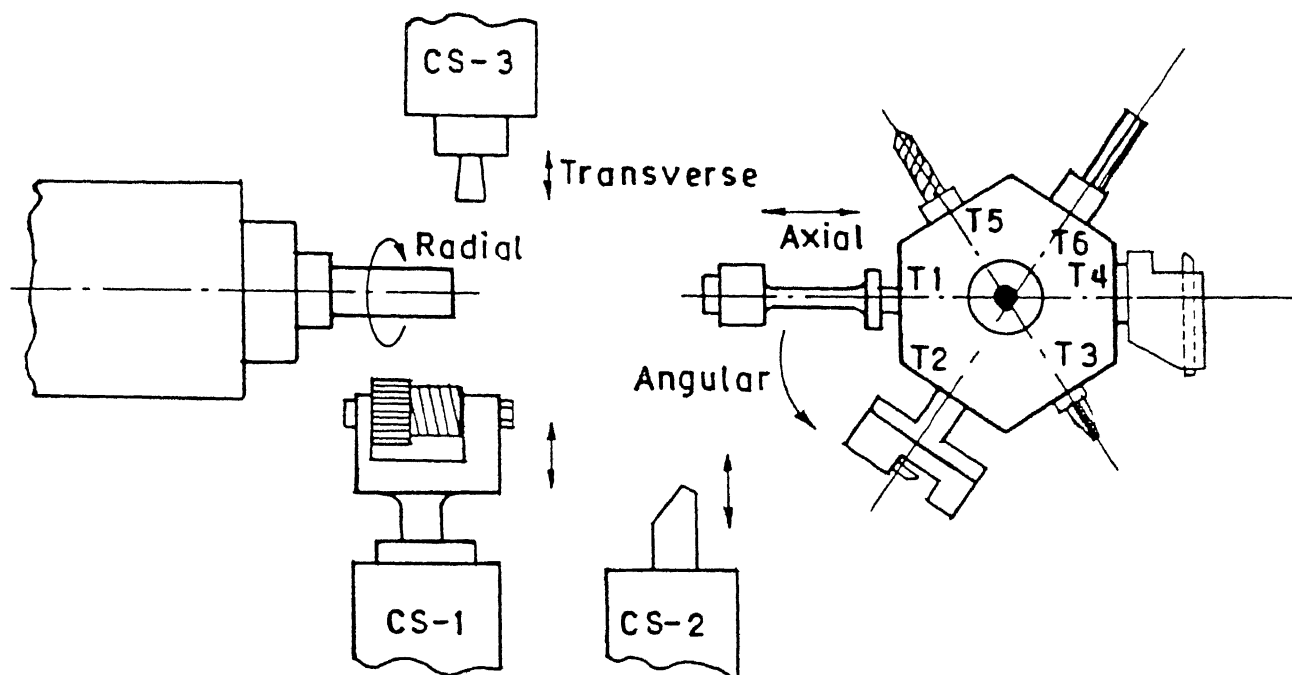


FIG.1.2 ILLUSTRATION OF RELATIVE MOTIONS IN AN AUTOMATIC MACHINE

TABLE 1.1

Simplified Work Dependent Input Data

Operations (Type of Motion Curves) M_i	Tool Travel S_i	Time (Cam Rotation Angle, at the End of Each Operation) β_i
M_1	S_1	β_1
M_2	S_2	β_2
M_3	S_3	β_3
\vdots	\vdots	\vdots
M_N^*	S_N	β_N

* N is the number of operations per workpiece.

of cam calculations [2] and modified constant velocity curves.

For modified constant velocity curves, which are basically used for cutting strokes, building block approach [3] has been adopted. This motion curve comprises of cycloidal motion for the beginning 10% and the ending 5% of each forward stroke and converse being for the withdrawal strokes.

For the radius of curvature calculations differential geometry approach has been adopted [1] which may be claimed as one of the best methods, as the evaluations of the same, becomes highly difficult through conventional evaluation. The package has been designed to avoid all kinds of confusions in the user while inputting the data. All kinds of necessary displays are given to the user allowing him at presignified points to go back for redesigning. The important data, such as radius of curvature, profile radius, pressure angle and cutter path (for specified cutter size) against cam angle are optionally displayed. The package also acts as an excellent tool in designing the plate cams to the specified limitations of radius of curvature and pressure angle. This also being optional, hints out to the user about the zone where actually the design is not within the specified limitations and leaves information as to how the correction can be made.

The cam display is strictly as per the conventions used in automatic machines [4, 5, 6, 7, 8, 9.] In this display the user can see normal vector, force vector and pressure angle vector at any point on the cam profile. It also contains optional display of the cutter path for a specified

cutter size. Following this main cam display the important plots of input parameters i.e., displacement, velocity and acceleration against cam angle is screened, with their respective magnitudes indicated optionally. Final display is about the output parameters i.e., radius of curvature, pressure angle, profile radius and contact point angle (with reference to arm carrying roller) in the same manner as of the previous plots with the above features the present work meets the requirements, which permits the work dependent plate cam design for automatic machines accurately, quickly and economically. As this was the existing need for the present day industries with sufficiently good computing and graphics facilities well within the reach, the present work has been taken up. Another motivating factor for aiming at this task is to take a step towards CAM (Computer Aided Manufacturing) in the fast developing modern industries.

1.4 Organisation of the Present Work

In Chapter 2 derivations relating to the contact condition, cam profile determination, pressure angle and radius of curvature have been dealt, with respect to a specified input acceleration, velocity, displacement of tool stations and cam modulated linkage used. It also contains a brief description about the motion curves used in developing the package.

Chapter 3 gives a clear flow picture of the package through a detailed flow chart and algorithms for some important modules. Following this there is a brief note about the interactive features embodied in the package.

In Chapter 4, the results have been analysed and discussed for some typical examples of workpieces which are manufactured on single spindle automata.

In the final chapter, the scope for further work and conclusions are presented.

The presentation contains three appendices. The Appendix I gives a typical example of a cam layout sheet [9], which has an operation worksheet and a tool layout. The Appendix II gives the listing of motion curves used in the package. The Appendix III tells about the principle of curvature from the perspective of differential geometry used for the present work.

CHAPTER 2

DESIGN PROCEDURE

Consider, initially the cam modulated linkage used for the turret and front cross slides with all the coordinate axes identified as shown in the Figure 2.1. Let $S_4(X_4, Y_4, Z_4)$ be the coordinates attached to the cam and $S_2(X_2, Y_2, Z_2)$ be that of the oscillating arm carrying the roller. Let θ_4 be the cam rotation angle, θ_2 the arm angle, r_f the follower roller radius, d the follower arm length and θ_{20} the initial arm angle.

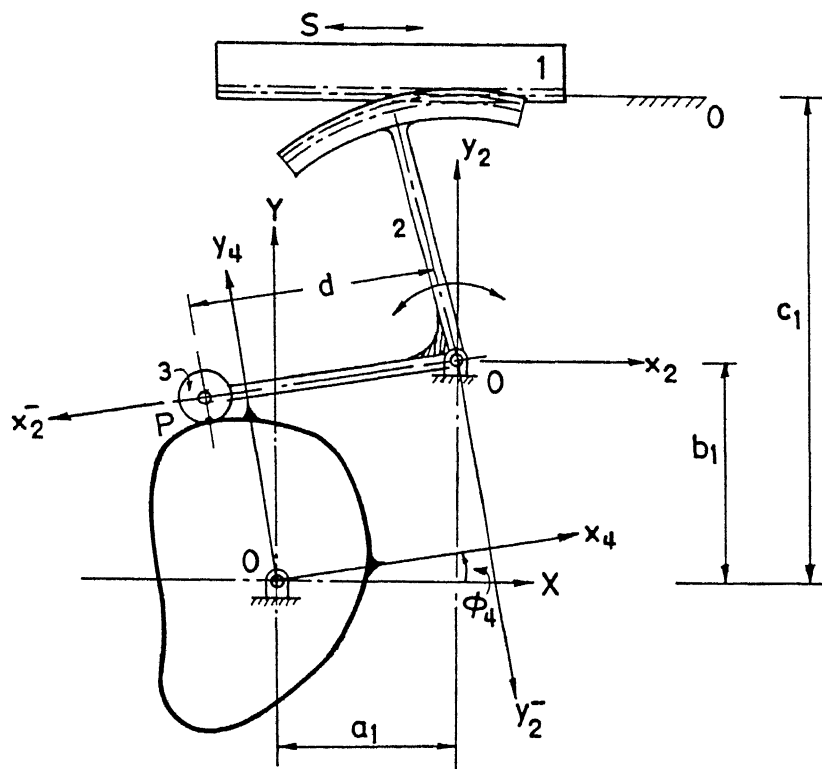
$S_2(X_2, Y_2, Z_2)$ is a coordinate system fixed at O_2 . With $S(X, Y, Z)$ as the global coordinate system, the rotating coordinates $S_4(X_4, Y_4, Z_4)$ and $S_2(X_2, Y_2, Z_2)$ are located. Let a_1 and b_1 are the distances from the arm hinge $O_2(X_2, Y_2, Z_2)$ and cam axis $O(X_4, Y_4, Z_4)$ along X and Y directions respectively. Consider S as the positive incremental displacement of the tool slide towards the work approaching direction and c_1 be the distance of the tool slide from the cam axis.

2.1 Profile Synthesis

For a specified angular velocity $\dot{\theta}_4 (= \omega_4)$ of the cam, slide displacement S , slide velocity V , the arm angle can be easily found as

$$S = (c_1 - b_1)(\theta_2 - \theta_{20}) \quad (2.1)$$

hence



14

FIG.2.1 ILLUSTRATION OF SYSTEM OF COORDINATE AXES WITH SYMBOLS

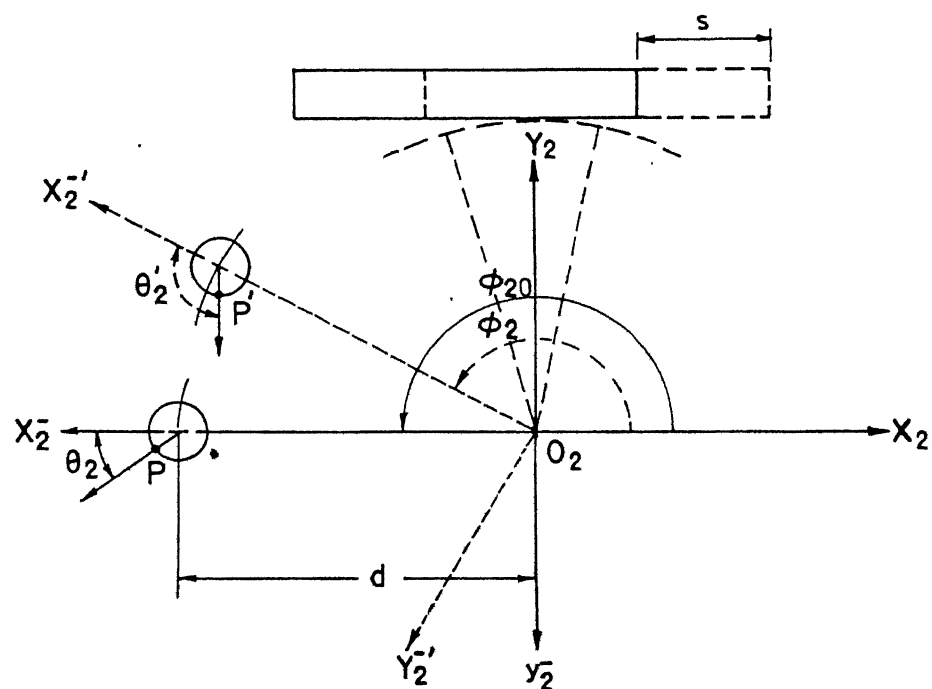


FIG.2.2 ILLUSTRATION OF ANGLES AND RELATIVE MOTION

$$\beta_2 = \beta_{20} + \frac{S}{(c_1 - b_1)} \quad (2.2)$$

and arm angular velocity $\beta'_2 = d\beta_2/d\beta_4$ becomes

$$\beta'_2 = \frac{1}{(c_1 - b_1)} S' \quad (2.3)$$

where $S' = dS/d\beta_4$, is nothing but the slide velocity V .

Figure 2.2 gives a clear picture of the system of coordinates and relative movements.

Let the cam rotate with a uniform velocity ω_4 , and let Ω_{24} be the uniform angular velocity of the follower.

Then it can be written

$$\Omega_{24} = \beta'_2 \times \omega_4 \quad (2.4)$$

Denoting the contact point direction, measured from the arm as θ_2 in the c.c.w.¹ direction as shown in Figure 2.2. The radius vector at any point P on the follower surface expressed in the fixed coordinate system is given by

$$\underline{r}^{(p_4)} = \begin{pmatrix} a_1 + d \cos \beta_2 + r_f \cos(\beta_2 + \theta_2) \\ b_1 + d \sin \beta_2 + r_f \sin(\beta_2 + \theta_2) \\ 0 \end{pmatrix} \quad (2.5)$$

2.1.1 Determination of contact condition

As per contact condition criterion [1], if $\underline{v}^{(42)}$ is a relative velocity vector between the cam and follower roller surface then

¹counter clockwise.

$$\underline{n} \cdot \underline{v}^{(42)} = 0 \quad (2.6)$$

where \underline{n} is a common normal to the surfaces.

The relative velocity vector $\underline{v}^{(42)}$ at the point P is nothing but

$$\underline{v}^{(42)} = \underline{v}^{(P_4)} - \underline{v}^{(P_2)} \quad (2.7)$$

where $\underline{v}^{(P_4)}$ is the velocity vector of the point P on the cam surface and $\underline{v}^{(P_2)}$ being the velocity vector of P on the follower surface

$$\text{From Equation (2.5) } \underline{r}^{(P_4)} = \begin{vmatrix} x_4 \\ y_4 \\ 0 \end{vmatrix}$$

The velocity vector $\underline{v}^{(P_4)}$ can then be written as

$$\underline{v}^{(P_4)} = \omega_4 \times \underline{r}^{(P_4)} \quad (2.8)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \dot{\beta}_4 \\ x_4 & y_4 & 1 \end{vmatrix}$$

$$\underline{v}^{(P_4)} = \begin{vmatrix} -\dot{\beta}_4 y_4 \\ +\dot{\beta}_4 x_4 \\ 0 \end{vmatrix} \quad (2.9)$$

where

$$x_4 = a_1 + d \cos \beta_2 + x_f \cos(\beta_2 + \theta_2) \quad (2.10)$$

$$y_4 = b_1 + d \sin \beta_2 + x_f \sin(\beta_2 + \theta_2)$$

Similarly the velocity vector at the contacting point P on the follower surface is given by

$$\underline{v}^{(P_2)} = \omega_{24} \times (\underline{r}^{(P_4)} - \underline{r}^{(O_2)}) \quad (2.11)$$

where

$$\underline{r}^{(O_2)} = \begin{vmatrix} a_1 \\ b_1 \\ 0 \end{vmatrix}$$

Using Equations (2.4) and (2.5)

$$\underline{v}^{(P_2)} = \omega_4 \begin{vmatrix} i & j & k \\ 0 & 0 & \beta'_2 \\ x_1 & y_1 & 1 \end{vmatrix}$$

i.e.,

$$\underline{v}^{(P_2)} = \omega_4 \begin{vmatrix} -\beta'_2 y_1 \\ +\beta'_2 x_1 \\ 0 \end{vmatrix} \quad (2.12)$$

where

$$x_1 = d \cos \beta_2 + r_f \cos(\beta_2 + \theta_2) \quad (2.13)$$

$$y_1 = d \sin \beta_2 + r_f \sin(\beta_2 + \theta_2)$$

The common normal \underline{n} between the cam and the follower surface is given by

$$\underline{n} = \begin{vmatrix} \cos(\theta_2 + \beta_2) \\ \sin(\theta_2 + \beta_2) \\ 0 \end{vmatrix} \quad (2.14)$$

Now, using the expression of relative velocity (2.7), we get

$$\underline{v}^{(42)} = \begin{vmatrix} -\dot{\beta}_4 y_4 + \beta_2^1 \omega_4 y_1 \\ \dot{\beta}_4 x_4 - \beta_2^1 \omega_4 x_1 \\ 0 \end{vmatrix}$$

As $\dot{\beta}_4 = \omega_4$

$$\underline{v}^{(42)} = \omega_4 \begin{vmatrix} -y_4 + \beta_2^1 y_1 \\ x_4 - \beta_2^1 x_1 \\ 0 \end{vmatrix} \quad (2.15)$$

Now applying condition of contact, i.e. Equation (2.6), we get the expression

$$\cos(\theta_2 + \beta_2)(-y_4 + \beta_2^1 y_1) + \sin(\theta_2 + \beta_2)(x_4 - \beta_2^1 x_1) = 0 \quad (2.16)$$

Now this equation has to be solved to get an expression for θ_2 which is the contact point direction measured from the positive x_2 -axis in c.c.w. direction, as shown in Figure 2.2.

Substituting the Equations (2.10) and (2.13) in (2.16), we get,

$$\begin{aligned} & (\beta_2^1 - 1) r_f [C\beta_2 S\beta_2 C^2\beta_2 + C^2\beta_2 \cos\theta_2 S\theta_2] + \\ & (1 - \beta_2^1) r_f [S^2\beta_2 C\beta_2 S\theta_2 + C\beta_2 S\beta_2 S^2\theta_2] + \\ & (1 - \beta_2^1) r_f [C^2\beta_2 S\theta_2 \cos\theta_2 - S\beta_2 C\beta_2 S^2\theta_2] + \\ & (1 - \beta_2^1) r_f [C\beta_2 S\beta_2 C^2\beta_2 - S^2\beta_2 C\beta_2 S\theta_2] + \\ & S\theta_2 \left\{ (b_1 S\beta_2 + a_1 C\beta_2 + d) - \beta_2^1 d(S^2\beta_2 + C^2\beta_2) \right\} + \\ & - \cos\theta_2 \left\{ (b_1 C\beta_2 + d S\beta_2 C\beta_2 + d S\beta_2 - d C\beta_2 S\beta_2) \right\} \\ & - \beta_2^1 (d S\beta_2 C\beta_2 - d S\beta_2 C\beta_2) = 0 \end{aligned}$$

It is to be noted that for the sake of convenience $\sin(\text{argument})$ and $\cos(\text{argument})$ are abbreviated as $S(\text{argument})$ and $C(\text{argument})$ respectively.

After simplification, the above equation reduces to

$$\left\{ (b_1 S\beta_2 + a_1 C\beta_2 + d) - \beta_2' d \right\} S\theta_2 - (b_1 C\beta_2 - a_1 S\beta_2) C\theta_2 = 0 \quad (2.17)$$

This equation can be written as,

$$E \sin\theta_2 - F \cos\theta_2 = 0$$

where

$$E = (b_1 S\beta_2 + a_1 C\beta_2 + d) - \beta_2' d \quad (2.18)$$

$$\text{and } F = (b_1 C\beta_2 - a_1 S\beta_2)$$

Hence from the equation of contact condition (2.17) the contact point direction θ_2 is obtained and is equal to

$$\theta_2 = \tan^{-1}(F/E) \quad (2.19)$$

Once this angle is determined the cam profile determination becomes very simple.

2.1.2 Determination of profile coordinates and pressure angle

After knowing contact point direction θ_2 , the equation for cam surface $R_4^{(P)}$ in $S(X, Y, Z)$ coordinate system is given by

$$R_4^{(P)} = N_{20} \left| R_2^{(P)} \right| \quad (2.20)$$

where $R_2^{(P)}$ is the equation of the radius vector of the contact point in the fixed coordinate system and is given by

$$R_2^{(P)} = \begin{vmatrix} d \cos \beta_2 + r_f \cos(\beta_2 + \theta_2) \\ d \sin \beta_2 + r_f \sin(\beta_2 + \theta_2) \\ 0 \\ 1 \end{vmatrix} \quad (2.21)$$

and M_{20} is coordinate transformation matrix from system $S_2(X_2, Y_2, Z_2)$ to system $S(X, Y, Z)$ and is given by

$$M_{20} = \begin{vmatrix} \cos \beta_4 & \sin \beta_4 & 0 & (a_1 \cos \beta_4 + b_1 \sin \beta_4) \\ -\sin \beta_4 & \cos \beta_4 & 0 & (-a_1 \sin \beta_4 + b_1 \cos \beta_4) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (2.22)$$

Substituting the Equations (2.21) and (2.22) in Equation (2.20), we get the cam profile equation

$$R_4^{(P)} = \begin{vmatrix} x \\ y \\ 0 \\ 1 \end{vmatrix}$$

$$R_4^{(P)} = \begin{vmatrix} \{a_1 + d \cos \beta_2 + r_f \cos(\beta_2 + \theta_2)\} \cos \beta_4 \\ \quad + \{b_1 + d \sin \beta_2 + r_f \sin(\theta_2 + \beta_2)\} \sin \beta_4 \\ - \{a_1 + d \cos \beta_2 + r_f \cos(\beta_2 + \theta_2)\} \sin \beta_4 \\ \quad + \{b_1 + d \sin \beta_2 + r_f \sin(\theta_2 + \beta_2)\} \cos \beta_4 \\ 0 \\ 1 \end{vmatrix} \quad (2.23)$$

Pressure angle determination is very important in these mechanisms as it is an index for power and motion transmission. The pressure angle for a higher-pair contact is

defined as the angle subtended between the direction of applied force on the driven member and the direction of the velocity vector at the contact point on the driving member [1]. Hence the direction of the applied force is along the unit common normal vector \underline{n} . Then the pressure angle ψ is found by using equations of the unit normal \underline{n} (2.14) and the unit velocity vector at point of contact P on the follower surface $\underline{v}_2^{(P)}$ (2.13)

$$= \cos^{-1}(\underline{n} \cdot \underline{v}_2^{(P)}) \quad (2.24)$$

$$= \cos^{-1}\{(d \cos \theta_2)/(d^2 + r_f^2 - 2d r_f \cos \theta_2)^{1/2}\} \quad (2.25)$$

2.2 Curvature Analysis

The parameter of the cam surface $R_4^{(P)}$ is the cam rotation angle β_4 .

We have already the unit normal vector at the point of contact on the follower surface as

$$\underline{n} = \begin{bmatrix} \cos(\theta_2 + \beta_2) \\ \sin(\theta_2 + \beta_2) \\ 0 \\ 1 \end{bmatrix}$$

The unit normal \hat{e}_4 on the cam surface is given by

$$\hat{e}_4 = [M_{24}] \cdot \underline{n} \quad (2.26)$$

where M_{24} is the simple rotational transformation matrix from $S_2(X_2, Y_2, Z_2)$ to the system $S_4(X_4, Y_4, Z_4)$, i.e.

$$[M_{24}] = \begin{vmatrix} C\phi_4 & S\phi_4 & 0 & 0 \\ -S\phi_4 & C\phi_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (2.27)$$

Hence, the Equation (2.27) becomes,

$$\hat{e}_4 = \begin{vmatrix} C(\theta_2 + \phi_2 - \phi_4) \\ S(\theta_2 + \phi_2 - \phi_4) \\ 0 \\ 1 \end{vmatrix} \quad (2.28)$$

Now from the concepts of differential geometry (Appendix III) the principal curvature for the cam surface is given by

$$\chi_{\phi_4} = \frac{N}{G} \quad (2.29)$$

where

$$N = -\hat{e}_{4\phi_4} \cdot R_{4\phi_4}^{(P)} \quad (2.30a)$$

$$G = R_{4\phi_4}^{(P)} \cdot R_{4\phi_4}^{(P)} \quad (2.30b)$$

where

$$\hat{e}_{4\phi_4} = \frac{\partial \hat{e}_4}{\partial \phi_4} \quad \text{and} \quad R_{4\phi_4}^{(P)} = \frac{\partial R_4^{(P)}}{\partial \phi_4} \quad (2.31)$$

We have expression of the normal unit vector on the cam surface at point P as

$$\hat{e}_4 = \begin{vmatrix} C\phi_4 C(\phi_2 + \theta_2) + S\phi_4 S(\theta_2 + \phi_2) \\ S\phi_4 C(\theta_2 + \phi_2) + C\phi_4 S(\theta_2 + \phi_2) \\ 0 \\ 1 \end{vmatrix}$$

On differentiating partially with respect to β_4 the expression becomes,

$$e_{4\beta_4} = \begin{vmatrix} f_1 - (\theta'_2 + \beta'_2) S(\theta_2 + \beta_2 - \beta_4) \\ -f_1 - (\theta'_2 + \beta'_2) C(\theta_2 + \beta_2 - \beta_4) \\ 0 \\ 1 \end{vmatrix} \quad (2.32)$$

Similarly in partially differentiating $R_4^{(P)}$ i.e., the Equation (2.23) with respect to β_4 , the following steps occur. Expanding the Equation (2.33) and rewriting,

$$R_4^{(P)} = \begin{vmatrix} [a_1 + dC\beta_2 + r_f(C\beta_2 C\theta_2 - S\beta_2 S\theta_2)] C\beta_4 \\ + [b_1 + dS\beta_2 + r_f(S\theta_2 C\beta_2 + C\theta_2 S\beta_2)] S\beta_4 \\ - [a_1 + dC\beta_2 + r_f(C\beta_2 C\theta_2 - S\beta_2 S\theta_2)] S\beta_4 \\ + [b_1 + dS\beta_2 + r_f(S\theta_2 C\beta_2 + C\theta_2 S\beta_2)] C\beta_4 \\ 0 \\ 1 \end{vmatrix} \quad (2.33)$$

Now partially differentiating with respect to β_4 yields

$$R_{4\beta_4}^{(P)} = \begin{vmatrix} -a_1 S\beta_4 + b_1 C\beta_4 + (1 - \beta'_2) d S(\beta_2 - \beta_4) \\ + r_f [S(\theta_2 + \beta_2 - \beta_4) - (\theta'_2 + \beta'_2) S(\theta_2 + \beta_2 - \beta_4)] \\ - a_1 C\beta_4 + b_1 S\beta_4 - (1 - \beta'_2) d C(\beta_2 - \beta_4) \\ + r_f [-C(\theta_2 + \beta_2 - \beta_4) + (\theta'_2 + \beta'_2) C(\theta_2 + \beta_2 - \beta_4)] \\ 0 \\ 1 \end{vmatrix} \quad (2.34)$$

$$\text{where } \theta'_2 = d\theta_2/d\beta_4 \text{ and } \beta'_2 = d\beta_2/d\beta_4 \quad (2.35)$$

Let

$$X_{e_4'} = \{1 - (\theta_2' + \beta_2')\} S(\theta_2 + \beta_2 + \beta_4)$$

and

$$Y_{e_4'} = -\{1 - (\theta_2' + \beta_2')\} C(\theta_2 + \beta_2 + \beta_4)$$

Then Equations (2.32) and (2.34) become

$$\hat{e}_{4\beta_4} = \begin{vmatrix} X_{e_4'} \\ Y_{e_4'} \\ 0 \\ 1 \end{vmatrix}$$

$$R_{4\beta_4}^{(P)} = \begin{vmatrix} -a_1 S\beta_4 + b_1 C\beta_4 + (1 - \beta_2')d S(\beta_2 - \beta_4) + r_f \cdot X_{e_4'} \\ -a_1 C\beta_4 - b_1 S\beta_4 - (1 - \beta_2')d C(\beta_2 - \beta_4) + r_f \cdot Y_{e_4'} \\ 0 \\ 1 \end{vmatrix}$$

Now the Equation (2.30a) for N becomes

$$\begin{aligned} -N &= a_1(S\beta_4 \cdot X_{e_4'} + C\beta_4 Y_{e_4'}) + b_1(S\beta_4 Y_{e_4'} - C\beta_4 X_{e_4'}) + \\ &\quad (1 - \beta_2')d \{C(\beta_2 - \beta_4) Y_{e_4'} - S(\beta_2 - \beta_4)X_{e_4'}\} - \\ &\quad r_f[X_{e_4'}^2 + Y_{e_4'}^2] \end{aligned} \quad (2.36)$$

where

$$(S\beta_4 X_{e_4'} + C\beta_4 Y_{e_4'}) = (\theta_2' + \beta_2' - 1) C(\theta_2 + \beta_2)$$

$$S\beta_4 Y_{e_4'} - C\beta_4 X_{e_4'} = (\theta_2' + \beta_2' - 1) S(\theta_2 + \beta_2)$$

$$C(\beta_2 - \beta_4)Y_{e_4} - S(\beta_2 - \beta_4)X_{e_4} = (\theta_2' + \beta_2' - 1)C\alpha_2$$

$$\text{and } X_{e_4}^2 + Y_{e_4}^2 = (\theta_2' + \beta_2' - 1)^2$$

Back substituting these expressions in (2.36) Equation, expression for N becomes

$$\begin{aligned} N &= -\hat{e}_{4\beta_4} \cdot R_{4\beta_4}^{(P)} \\ &= (\theta_2' + \beta_2' - 1) \left[a_1 C(\theta_2 + \beta_2) + b_1 S(\theta_2 + \beta_2) + (1 - \beta_2')d C\alpha_2 \right. \\ &\quad \left. - r_f(\theta_2' + \beta_2' - 1) \right] \end{aligned} \quad (2.37)$$

Now taking the expression (2.30b), i.e.,

$$G = R_{4\beta_4}^{(P)} \cdot R_{4\beta_4}^{(P)}$$

the evaluation continues as follows,

$$\begin{aligned} R_{4\beta_4}^{(P)} \cdot R_{4\beta_4}^{(P)} &= r_f^2(X_{e_4}^2 + Y_{e_4}^2) + (1 - \beta_2')^2 d^2 + b_1^2 C^2 \beta_4 + \\ &\quad a_1^2 S^2 \beta_4 - 2a_1 b_1 C\beta_4 S\beta_4 + b_1^2 S^2 \beta_4 + a_1^2 C^2 \beta_4 + \\ &\quad 2a_1 b_1 S\beta_4 C\beta_4 + 2(1 - \beta_2')d \left[b_1 S(\beta_2 - \beta_4)C\beta_4 - \right. \\ &\quad \left. a_1 S(\beta_2 - \beta_4)S\beta_4 + b_1 C(\beta_2 - \beta_4)S\beta_4 + a_1 C(\beta_2 - \beta_4)C\beta_4 \right] \\ &\quad - 2r_f(1 - \beta_2')d \left\{ -S(\beta_2 - \beta_4)X_{e_4} + C(\beta_2 - \beta_4)Y_{e_4} \right\} \\ &\quad - 2r_f \left[a_1 (S\beta_4 X_{e_4} + C\beta_4 Y_{e_4}) + b_1 (S\beta_4 Y_{e_4} - C\beta_4 X_{e_4}) \right] \end{aligned}$$

After simplification, the above equation reduces to

$$G = (1 - \beta_2')^2 d^2 + a_1^2 + b_1^2 + 2(1 - \beta_2')d(b_1 S\beta_2 + a_1 C\beta_2) \\ - r_f^2(\theta_2' + \beta_2' - 1)^2 - 2r_f(\theta_2' + \beta_2' - 1) [a_1 C(\theta_2 + \beta_2) \\ + b_1 S(\theta_2 + \beta_2) + (1 - \beta_2')d C\theta - r_f(\theta_2' + \beta_2' - 1)]$$

which can be rewritten as,

$$G = K_{55} - 2r_f \cdot N \quad (2.38)$$

where

$$K_{55} = (1 - \beta_2')^2 d^2 + a_1^2 + b_1^2 + 2(1 - \beta_2')d(b_1 S\beta_2 \\ + a_1 C\beta_2) - r_f^2(\theta_2' + \beta_2' - 1)^2 \quad (2.39)$$

Hence principal curvature becomes,

$$\chi_{\beta_4} = \frac{N}{K_{55} - 2r_f \cdot N} \quad (2.40)$$

As the radius of curvature of the cam surface along β_4 lines is nothing but the reciprocal of principal curvature

$$\text{Radius of curvature } \rho = \frac{K_{55}}{N} - 2r_f \quad (2.41)$$

The expression for θ_2' used for the derivation of the radius of curvature is given below.

Rewriting the Equation (2.19) of θ_2 , the contact angle

$$\theta_2 = \tan^{-1}(F/E)$$

i.e.,

$$\theta_2 = \tan^{-1} \left\{ (b_1 C\beta_2 + a_1 S\beta_2) / (b_1 S\beta_2 + a_1 C\beta_2 + d - \beta_2' d) \right\}$$

$$\text{Now } \theta_2' = \frac{d\theta_2}{d\beta_2} = \frac{1}{\beta_4} \tan^{-1}(F/E)$$

$$\theta'_2 = \frac{1}{1 + (F/E)^2} * \frac{d}{d\theta_4} (F/E) \quad (2.42)$$

where

$$\frac{d}{d\theta_4} (F/E) = \frac{E \frac{dF}{d\theta_4} - F \frac{dE}{d\theta_4}}{E^2}$$

The numerator in the R.H.S. of the above equation reduces to

$$E \frac{dF}{d\theta_4} - F \frac{dE}{d\theta_4} = \theta'_2(\theta'_2 - 1)d(b_1 S\theta_2 + a_1 C\theta_2) - \theta'_2(a_1^2 + b_1^2) + \theta''_2 d(b_1 C\theta_2 - a_1 S\theta_2) \quad (2.43)$$

After all substitutions, the final expression for θ'_2 becomes,

$$\theta'_2 = \frac{\theta'_2(\theta'_2 - 1)d(b_1 S\theta_2 + a_1 C\theta_2) - \theta'_2(a_1^2 + b_1^2) + \theta''_2 d(b_1 C\theta_2 - a_1 S\theta_2)}{E^2 + F^2} \quad (2.44)$$

where, $\theta''_2 = \frac{d^2}{d\theta_4^2} (\theta_2)$

Substituting the equation for θ_2 i.e. (2.2)

$$\theta''_2 = \frac{1}{(c_1 - b_1)} S'' = \frac{1}{c_1 - b_1} A \quad (2.45)$$

where A is the acceleration of the tool slide.

So far the equations derived for profile coordinates, pressure angle and radius of curvature have been referred to the cam modulated linkage used for turret head and front cross slide movement.

In order to make them applicable for rear cross slide cam modulated linkages, shown in Figure 2.3, slight modifications have to be applied. They are as follows:

In this cam modulated linkage the initial angle of the arm is calculated by the expression

$$\beta_{20} = \text{PI} + \tan^{-1} \left\{ (b_2 - b_1)/(a_2 - a_1) \right\} + \text{Resetting angle} \quad (2.46)$$

This resetting angle can be set in the follower to suit the need.

Once the lengths a_1 , a_2 , b_1 and b_2 are known, the lengths of the links carrying the gear sectors e_1 and f_1 for a given ratio (of length f_1 to the sum of the lengths e_1 and f_1) are determined using simple geometry as follows. We have,

$$\text{Ratio} = f_1/(e_1 + f_1) \quad (2.47)$$

where $(e_1 + f_1)$ can easily be calculated by the expression

$$e_1 + f_1 = \left[(b_2 - b_1)^2 + (a_2 - a_1)^2 \right]^{1/2} \quad (2.48)$$

For the purpose of maintaining almost all symbols as in the first case of the turret and front cross slide cam modulated linkage, the naming of the link and other lengths have been chosen in this case, which is very clear from the Figure 2.3.

The expression for instantaneous arm angle is

$$\beta_2 = \beta_{20} + \frac{S}{(e_2 - b_2)} (f_1/e_1) \quad (2.49)$$

Also

$$\beta_2' = \frac{V}{(e_2 - b_2)} (f_1/e_1) \quad (2.50)$$

and

$$\beta_2'' = \frac{A}{(e_2 - b_2)} (f_1/e_1) \quad (2.51)$$

where S , V and A are displacement, velocity and acceleration of the rear cross slide.

Bringing the above modifications, all other equations for the contact condition, profile calculations, pressure angle calculation, curvature calculations hold good with the symbols as presented in the first case of turret or front slide cam.

2.3 Cutter Coordinates Determination

To produce a right cam profile, the location of the milling cutter or of the grinding wheel should be known. Let r_c be the radius of the cutter, then to find the cutter coordinates (x_c, y_c) or the cutter path envelope $R_c^{(c)}$. Consider, the unit normal vector to the cam surface given by Equation (2.28) and the cam profile Equation (2.23). Then,

$$R_c^{(c)} = R_4^{(P)} + r_c \hat{e}_4 \quad (2.52)$$

By substituting the corresponding expressions of $R_4^{(P)}$ and \hat{e}_4 the cutter coordinate is determined, i.e., Equation (2.31) can also be written as

$$\begin{vmatrix} x_c \\ y_c \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} x \\ y \\ 0 \\ 1 \end{vmatrix} + r_c \begin{vmatrix} c(\theta_2 + \beta_2 - \beta_4) \\ s(\theta_2 + \beta_2 - \beta_4) \\ 0 \\ 1 \end{vmatrix} \quad (2.53)$$

2.4 Summary About Motion Programs Adopted in the Software

Motion curves are the operation dependent inputting data. They precisely describe the motion of the tool during

working and idle strokes. Depending upon the operation to be performed on the workpiece, a specific pattern of motion parameters expressed against time, has to be decided. The motion parameters are nothing but the displacement, velocity and acceleration of the follower driven tool station. Each set of these motion curves are generally described in a motion program to carry out the operation of desired nature.

One of the main requirements for the design and manufacture of cams for automatic machines is to process a set of motion curves from which any complex acceleration pattern can be obtained by using 'building block' approach or by simple augmentation of elementary and essential trigonometric and modified trigonometric curves.

In the present work, for the most commonly used cutting operations such as turning, thread cutting etc., the modified constant velocity curves have been developed. For the rest of the operations such as forming, indexing, parting etc., modified trapezoidal curves, modified sinusoidal, and sinusoidal curves have been employed. The Appendix III contains, the plots of all these curves and the equations for the corresponding rise motions. The motion curves, other than modified constant velocity curves, have been derived keeping 'universal' system of cam calculations for automatic machines as per reference [2].

The modified constant velocity curve developed in the present work is a combination of cycloidal and constant velocity curves. For the rising stroke, i.e., for modified

constant velocity rise motion, first 10% of the total slide travel is the rising cycloidal velocity motion, followed by the next 85% of the total stroke, an uniform velocity motion and the remaining 5% of the stroke is a falling cycloidal velocity motion. Similarly the modified constant velocity fall (MCVF) curve contains the combination in the reverse order.

The remaining category of motion curves have been developed, under three basic types of movements named as, Type I, Type II and Type III

Type I - refers to a motion having a strokes starting from a dwell and ending in a dwell.

Type II - has the forward stroke followed by the return stroke, which in turn is followed by the next forward stroke, hence, without any dwell.

Type III - has the forward stroke starting from a dwell, followed by a return stroke without any dwell in between them and at the end. This is a combination of Type I and Type II.

Figure 2.4 shows Types I, II and III acceleration curves with the values of coefficients of maximum acceleration and maximum velocity, i.e., C_A^1 and C_V^2 used in developing, the modified

¹ C_A , the coefficient of maximum acceleration represents the maximum value acceleration when the ratio of stroke (S) to square of the time duration (T) of the stroke is unity, i.e. when $S/T^2 = 1$.

² C_V , the coefficient of maximum velocity represents the maximum value of the velocity when the ratio of stroke (S) to the time duration (T) in performing the stroke is unity, i.e., $S/T = 1$. Physically these coefficients indicate, the magnitude of the maximum values of the velocity or acceleration is comparison to their average values.

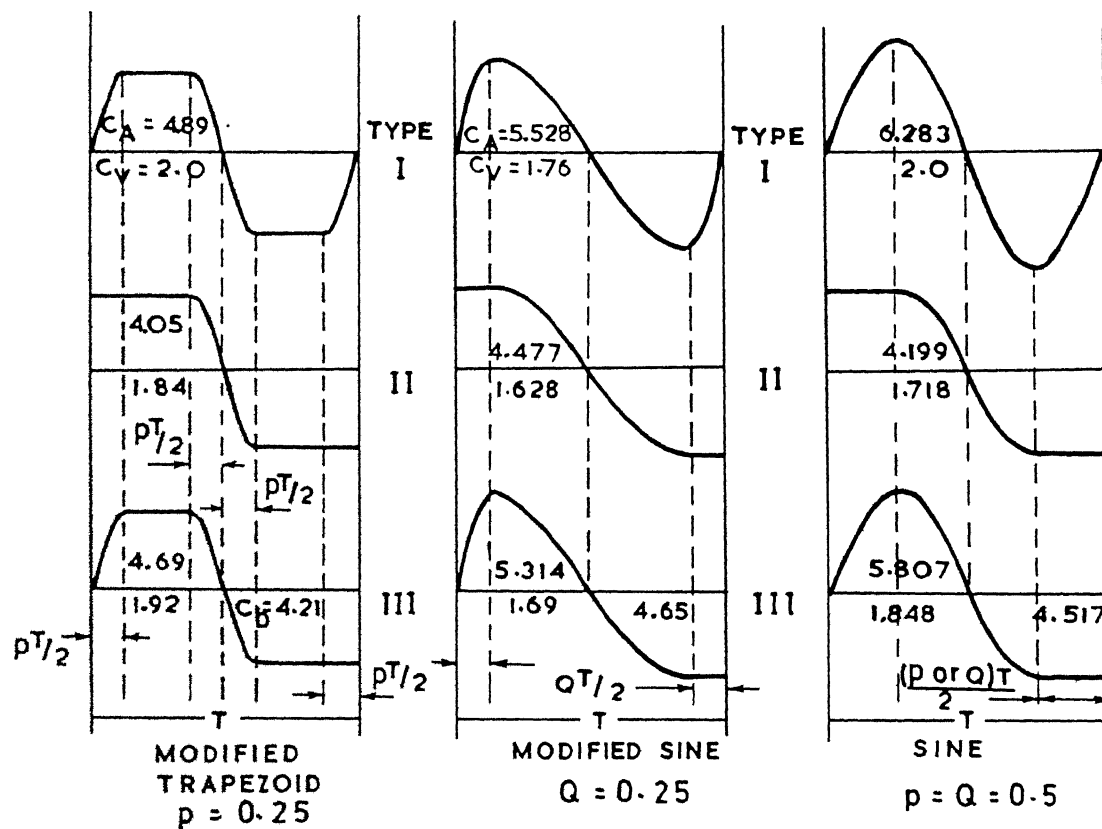


FIG. 2.4 ILLUSTRATION OF TYPE I II & III CURVES

trapezoidal, modified sinusoidal and sinusoidal motion curves in the package. Modified trapezoid group of curves are preferred when a lower acceleration is important or when higher C_v is desirable, which requires a cam follower system with high frequency of vibration.

When a high output torque is required, modified sine curves are preferred, for example, modified sine curves can be used for indexing operations. Sine group of curves are employed when the speed is high and in the case of insufficient stiffness of cam modulated linkage.

At the outset, the selection of these motion programs, developed in the package depends mainly upon the nature of the operation to be performed on the workpiece.

CHAPTER 3

SOFTWARE DEVELOPMENT

3.1 Implementation Details

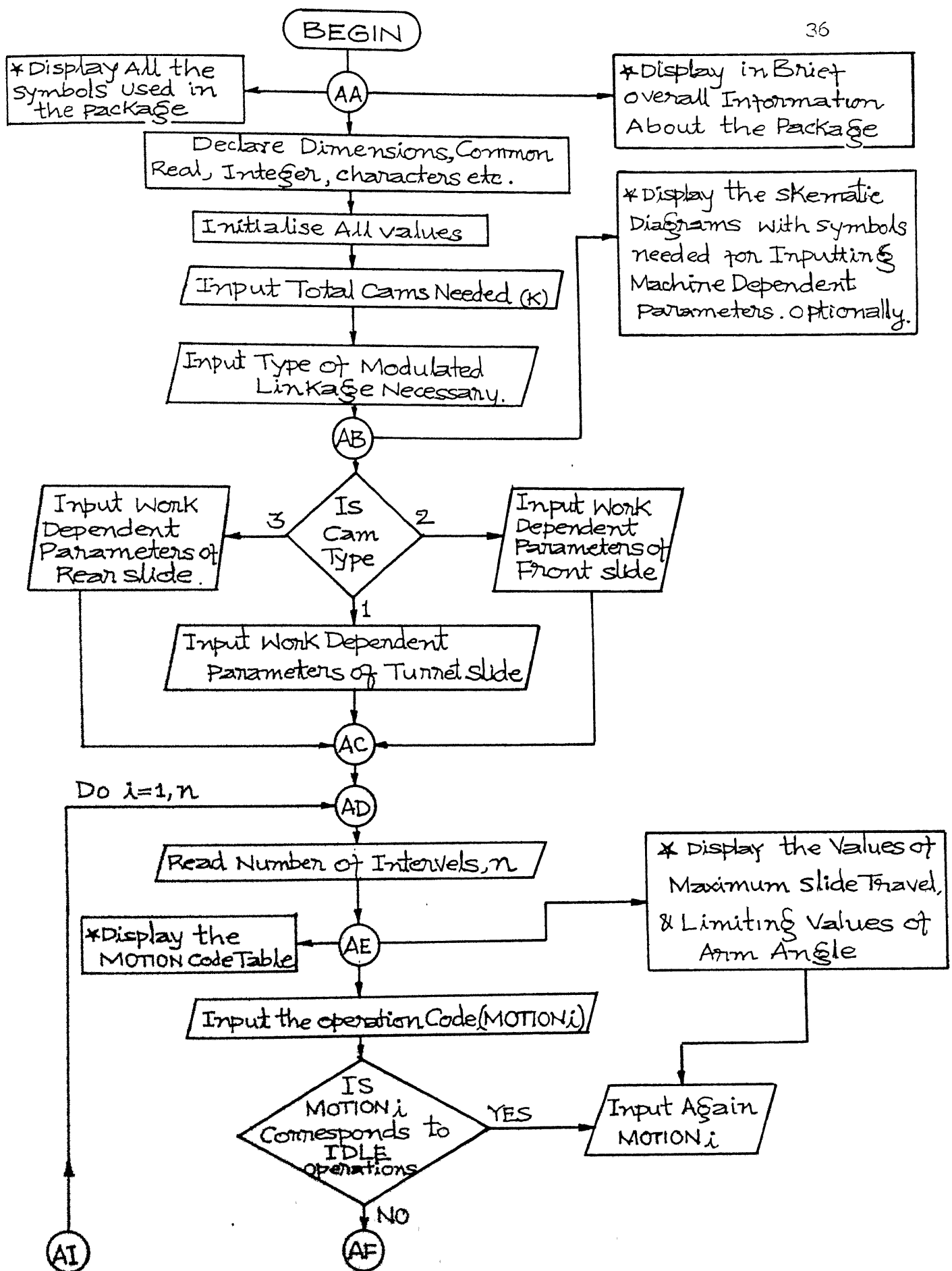
The present software has been implemented on Norsk Data (Norway)'s 32-bit Super-Mini Computer (i.e., ND-560/CXA) system. The programs for the package have been written in the language ANSI Fortran-77 with ND Extensions. For graphic displays the Tektronix 4109 and 4107 colour graphics terminals are used, and graphic primitives have been obtained using setup (local) commands of Tektronix.

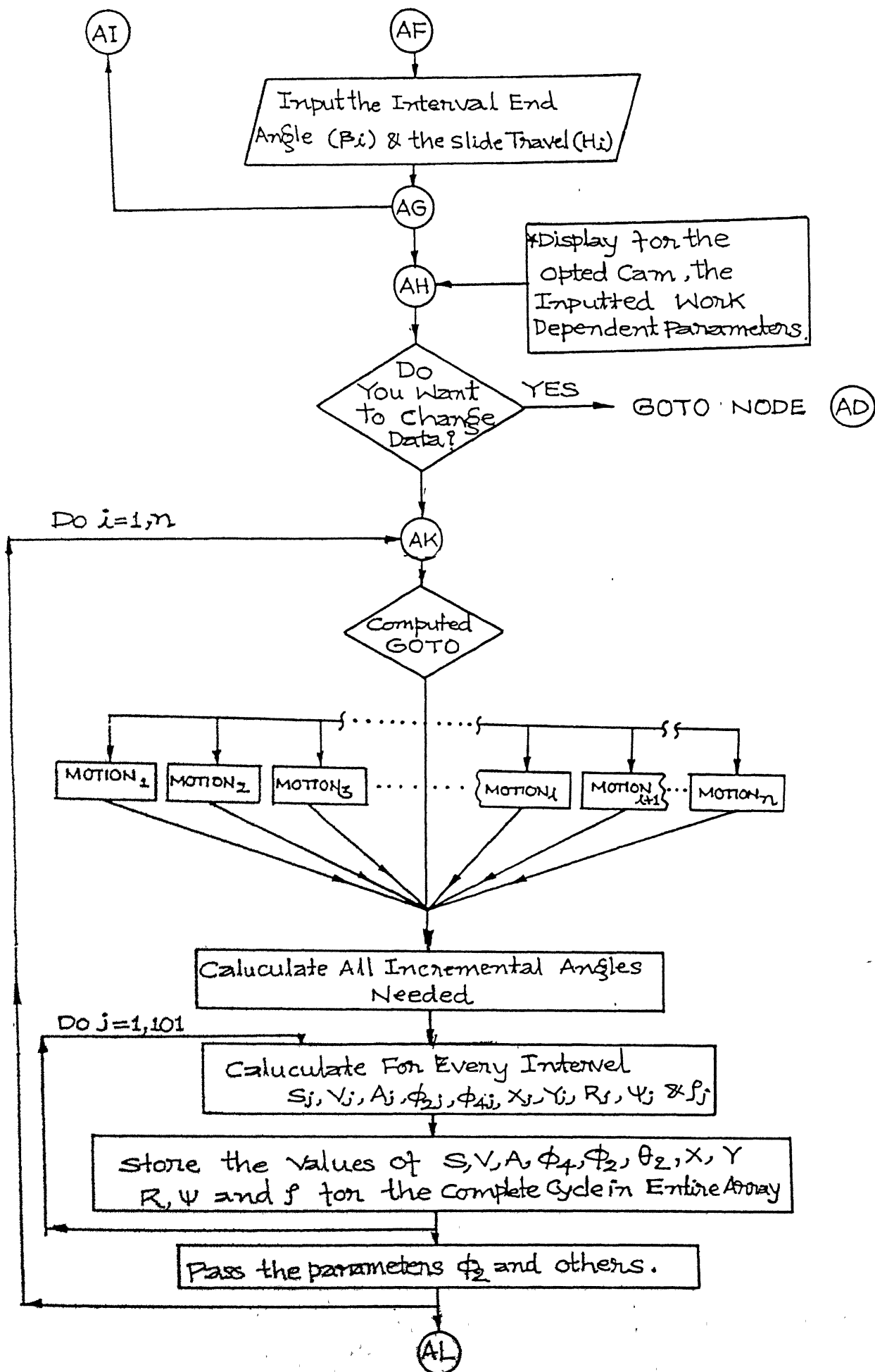
The following sections, discuss the details of the software, although the package has partial self-documentation features like, meaningful symbols and comments at suitable points.

3.2 Program Structure

The flowchart of the entire software is shown in the Figure 3.1. The symbols and conventions for the flowchart and other finer details in developing the software have been extracted from the references [10, 11, 12, 13] and the ND Fortran Reference Manual. References [14, 15] were essential in getting fundamental ideas about graphical primitives implementation.

The program starts with the declaration of all parameters, variables, characters and constants. Then the





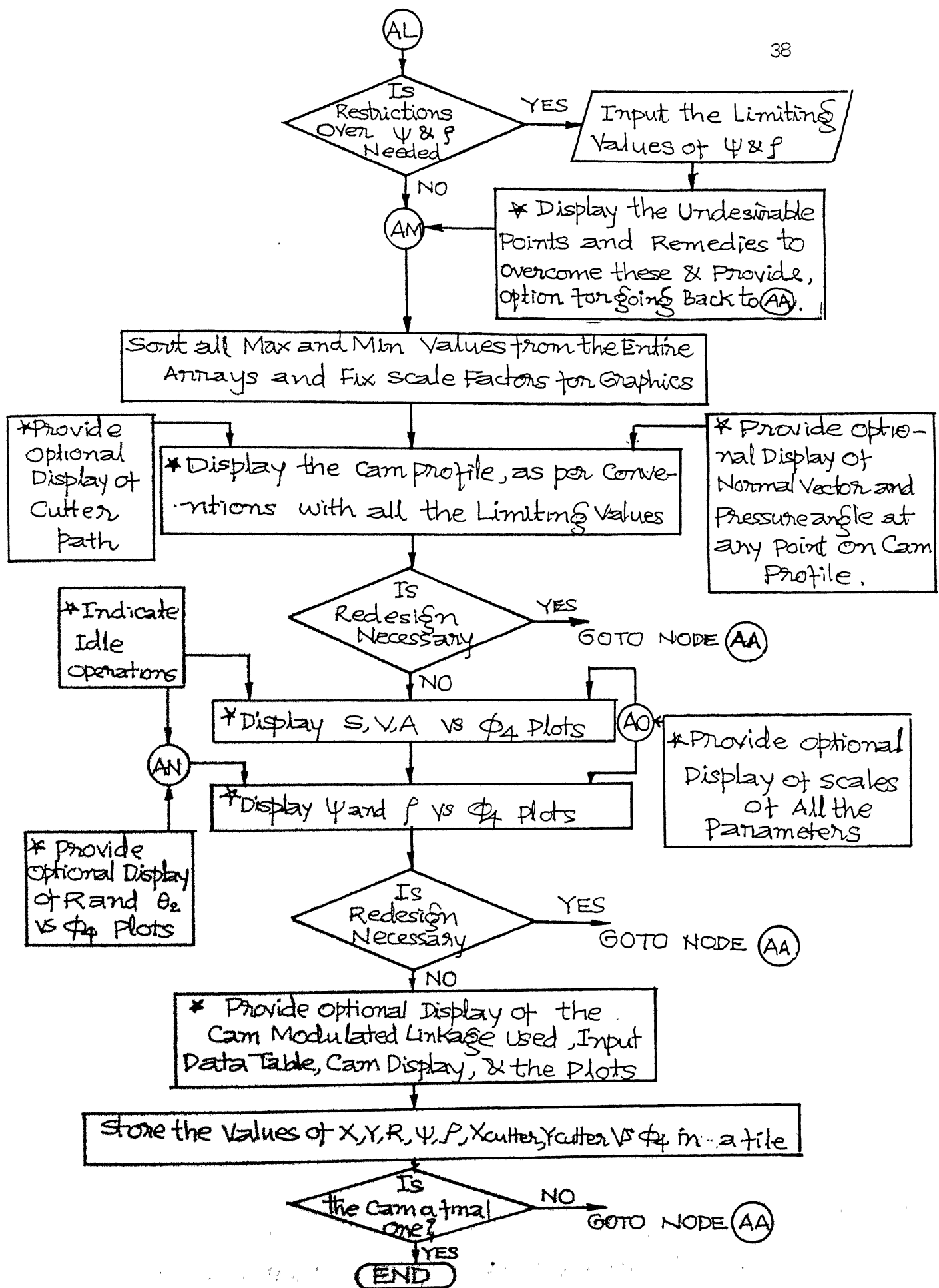


FIG. 3.1 Flow chart of the Entire Software.

subroutines containing the summary of the package, symbols used and other informations are called. The next module calls the User, to input the data pertaining to the work dependent and Machine dependent variables. In this module, all helpful displays to feed the data are contained.

When feeding of the data is completed, at Node (AH) the program displays the inputted values of work dependent parameters and other informations like the number of cams the User wants to design, the number of intervals the User has selected and the current type of cam with its number. At this stage the User can go for refeeding of the data, optionally. In the next module of the program, the files containing the normalised values of the displacement, velocity and acceleration for the motion curves are opened for reading using computed GO TO as shown in the flowchart. This module begins from the node (AK) and ends at the node (AL) having two nested Do loops. The outer Do loop is for one to number of intervals and the inner loop is for calculating the incremental values of all the parameters for a chosen number of 101 points. Hence for every interval, depending upon the motion curve selected, the particular data file which contains the normalised values of the displacement velocity and acceleration for 100 incremental steps (or 101 points) is read. Then in the inner loop, simple calculations are done to get the actual displacement, velocity and acceleration. Thus doing this way, has considerably reduced the computation time when compared to performing all computations and calculations in this module only, by

invoking subroutines. This module does all angle calculations, calculates and stores the contact points, cam profile coordinates incremental arm angle, incremental cam angle profile radii, incremental pressure angle, and radii of curvature and cutter coordinates for the entire cycle of the selected cam.

The next module is from the node (AL) to (AM) where the User can impose optionally, the restrictions on the maximum pressure angle and minimum radius of curvature. When opted for designing the cam within the specified limitations mentioned above, the program displays where actually the values are undesirable and tells what should be done to correct the same. At this stage again the program provides option in redesigning the cam by going back to node (AB) or to continue.

The next part of the program contains sorting of all maximum and minimum values of the essential parameters which are stored in arrays and finding out scale factors for plotting all the curves used in the package.

The next module of the program is the graphical output area (shown as dotted enclosure in the flowchart) which does the following operations:

- Displays, the designed cam profile as per conventions, on one side of screen and shows all necessary values of maximum and minimum values of parameters of interest.
- Provides an optional display of the normal vector, force vector and pressure angle at any angle on the cam profile.

- Provides an optional display of cutter path. After this display, a provision has been kept for the User to opt for redesigning.

- Next display is about the plots of displacement, velocity and acceleration against the cam rotation angle, with the optional displays of the respective scales. If the cam is for turret slide, then the User can recognise the idle operations, feed bar stop, indexing and clearing off of the tools.

- Following the above display of the plots, pertaining to inputted motion parameters, the next display is about the output variations of pressure angle and radius of curvature, with optionally displayed variations of profile radius and contact point angle, all being shown against cam rotation angle exactly in a similar fashion as that of the previous one. After displaying the above plots the programs can take the user to the starting node (AB) if desired.

- Finally in this subsection of the graphical output area all the displays right from the beginning are shown one after the other and terminates or displays again all of them, as per the User's option. With the above display, the graphical presentation concludes for the considered cam.

Lastly, the important output data pertaining to profile coordinates, profile radius, pressure angle, radius of curvature, cutter coordinates against the corresponding cam rotational angle are written down in a separate file for the purpose of analysis, and manufacture of the designed cam.

The program finally reaches the end, if the currently designed cam is being the last one in list, else the cycle of computer aided designing starts again.

3.3 Some Important Algorithms

While implementing the software most of the variable names as they appear in the design equations have been retained. However in this section, only two algorithms for cam parameters evaluation (CAMPEVA) and motion program (MOTION) are given in view of better understanding of the software. The algorithms are written as per the reference [11].

Algorithm CAMPEVA (CAM Parameters EVALuation): Given the angles at the end of each interval β_i , slide travel h_i and motion type $MOTION_i$ for the flag stations, i such that, $i = 1, n$, where n represents the number of intervals, the aim is to compute the profile coordinates X_j, Y_j , the pressure angle ψ_j and the radius of curvature ρ_j . The suffix j is from 1 to 101 as 100 subdivided are chosen within an interval. For the evaluation of these parameters the cam angle β_{4j} , the contact point angle θ_{2j} , the arm angle β_{2j} , and the input motion parameters displacement S_j , velocity V_j and acceleration A_j are to be computed. For finding out the actual motion parameters S_j, V_j and A_j the normalised motion parameters¹

¹Normalised motion parameters are those, which are calculated for unity rise or fall with the interval periods also as

SO_j , VO_j and AO_j are used. The values of SO_j , VO_j and AO_j are calculated using the algorithm MOTION and are stored in files of unit numbers U_m where m is from 1 to l , l being the number of motion program data files used in the software.

Step 0. [Initialise the K_{tot} array and open all data files for reading] $K_{tot} \leftarrow 0.0$

Step 1. For $i \leftarrow 1$ to n do step $i+1$

Step A. [Motion curve selection, using computed MOTD] Case 1 of $U_1; U_2; U_3; \dots U_m \dots U_l$ (where U stands for the file unit number)

Step $K+1$. [Rewind and read the file U_i] Read the normalised values SO ; VO and AO for $MOTION_i$ (Here SO , VO , AO are the arrays, each containing 101 values)

Step $K+2$. [Calculation of incremental angles]
If $i \neq 1$ then set $IntBeg_i \leftarrow \beta_{i-1}$; and $IntAng_i \leftarrow \beta_i - \beta_{i-1}$; else Set $IntBeg_i \leftarrow 0.0$; and $IntAng_i \leftarrow \beta_i$; end if ($IntBeg_i$ is the interval beginning angle and $IntAng_i$ is the interval angle)

Step $K+3$. Set $Temp \leftarrow IntAng_i / 100$

Step $K+4$. For $j \leftarrow 1$ to 101 do step $j+1$

Step $K+5$. If $j \neq 1$ then $\beta_{4j} = \beta_{4j-1} + Temp$; else $\beta_{4j} = 0$; end if

Step $K+6$. Set $\beta_{4j} \leftarrow IntBeg_i + \beta_{4j}$

Step K+7. Set $S_j \leftarrow SO_j * h_1$;
 $V_j \leftarrow VO_j * h_1 / \beta_1$
 $A_j \leftarrow AO_j * h_1 / (\beta_1 ** 2)$

Step K+8. [Calculate ϕ_{2j} , ϕ'_{2j} and ϕ''_{2j} etc.] (where ϕ'_{2j} and ϕ''_{2j} are first and second derivatives of ϕ_{2j} with respect to ϕ_{4j})

Step K+9. [Using contact condition find contact point direction Θ_{2j}]

Step K+10. [Evaluate the profile coordinates X_j and Y_j]

Step K+11. [Calculate pressure angle ψ_j]

Step K+12. [Calculate radius of curvature ρ_j]

Step K+13. $K_{tot} \leftarrow K_{tot} + 1$

Step K+14. Set $S_{K_{tot}} \leftarrow S_j$;

$V_{K_{tot}} \leftarrow V_j$;

$A_{K_{tot}} \leftarrow A_j$;

$X_{K_{ent}} \leftarrow X_j$;

$Y_{K_{ent}} \leftarrow Y_j$;

$\psi_{K_{ent}} \leftarrow \psi_j$;

$\rho_{K_{ent}} \leftarrow \rho_j$ (where

$K_{tot} = 1$ to K_{entire} and

thereby $K_{entire} = n \times 101$)

Step K+15. and do

Step K+17. [Pass parameters]

Step K+18. end do

Step K+19. stop

Step K+20. end

This algorithm finally contains all the values of essential parameters S , V , A , \dot{V} , Y , ψ and f stored in arrays each of dimension K_{entire} .

Algorithm MOTION: In this algorithm the normalised values of motion characteristics SO_j , VO_j and AO_j are calculated and writes them in a data file for the chosen acceleration, velocity and displacement characteristics.

Step 0. [Initialise] $X \leftarrow 0.0$

Step 1. For $j \leftarrow 1$ to 101 do step j+1

Step 2. Set $\gamma_j \leftarrow X/100$;

$X \leftarrow X+1$

Step 3. If $\gamma_j \leq \text{condition 1}$ then

Step 4. Set $SO_j \leftarrow \text{expression 1}$;

$VO_j \leftarrow \text{expression 2}$;

$AO_j \leftarrow \text{expression 3}$

Step 5. Else if $\gamma_j \leq \text{condition 1}$ and;
 $\gamma_j \leq \text{condition 2}$ then;

Set $SO_j \leftarrow \text{expression 4}$

$VO_j \leftarrow \text{expression 5}$

$AO_j \leftarrow \text{expression 6}$

Step 6+KN. End if (where KN depends on
the number of conditions)

Step 7+KN. End do

Step 8+KN. [Open sequential unformatted files for
writing the values of SO, VO and AO
arrays]

Step 9+KN. [Rewind the unit and write]

Step 10+KN. end

3.4 Interactive Features and Getting Used to the Package

The extensive use of Tektronix local commands has helped in providing a better graphical output communication to the user. There is a dialog buffer facility incorporated in the software, by which the user can go back and see the outputted messages and the inputted datas anywhere during execution, using the Joy disc control. The package has, at some essential points in the program, the facility of transferring the control to the starting point for redesigning. These return controls are indicated clearly in the Figure 3.1. Below are enlisted, a brief description about the User-friendly interaction messages and other displays, in the order of their occurrence during execution. These enlisted displays are pertaining to:

- the general information about the package.
- the essential variable and constant names used.
- the type of cam modulated linkage used. Figures 3.2, 3.3 and 3.4 show the photographs of these displays. The objective of these displays are to enable the User, to feed Machine Dependent data. It also asks the User to feed the total cam numbers to be used.

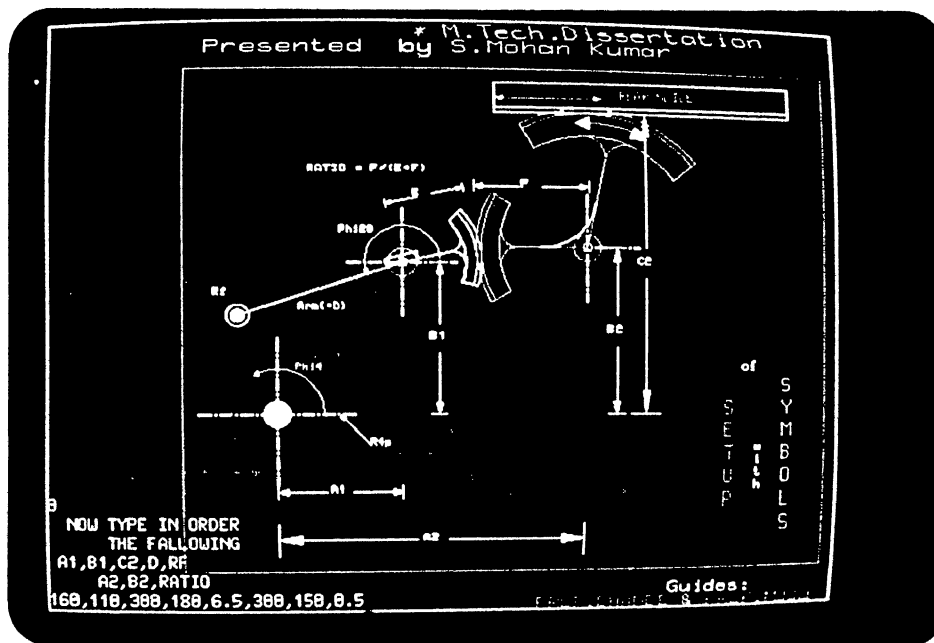


Fig. 3.4 Photograph Showing the Display of Rear Slide Cam Modulated Linkage

- the inputting of the work dependent data. This section starts asking the user all the details pertaining to the work dependent data like the number of intervals chosen, the type of motion curve needed or the type of operation to be performed, the slide travel in mms relating to Rise, Fall and Dwell and the cam angle at the end of each interval or operation. In order to feed the type of motion curve for the working operations and the idle operations, the display of the motion table with code number is given, from which the User is required to pick code number and type. During the above feeding of the work dependent data, at each interval the current status will be indicated. Figure 3.5 shows the photograph of one such displays.
- the fed work dependent inputs in a tabular form, wherein the user can go back for refeeding of the data in case of typing errors. Figure 3.6 shows the photograph of one such displays.

After the feeding of the data is completed, then the output displays are screened one by one as per the user's options, which are explained clearly while discussing the flowchart in the last section. During displaying the cam designed, the Normal vector, the Force vector and the Pressure angle which are optionally displayed at any point on the profile, seem to be very small due to the screen restriction. This can be overcome by doing 'zooming' and 'panning' operation by going into the Menu. Similarly in the case of radius of curvature

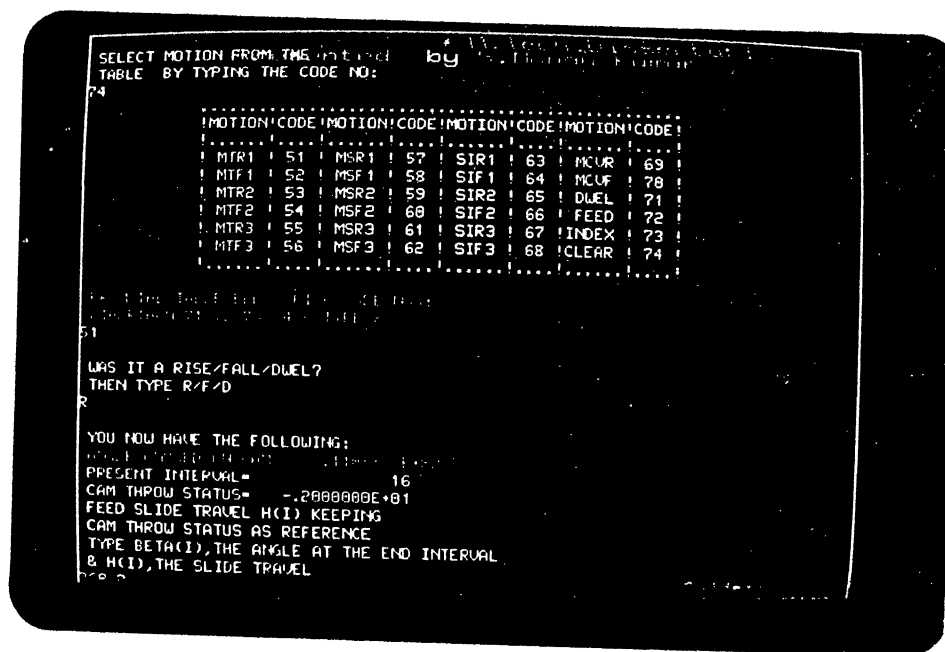


Fig. 3.5 Photograph Showing the Display of Motion Code Table with Indication of Status at Every Interval

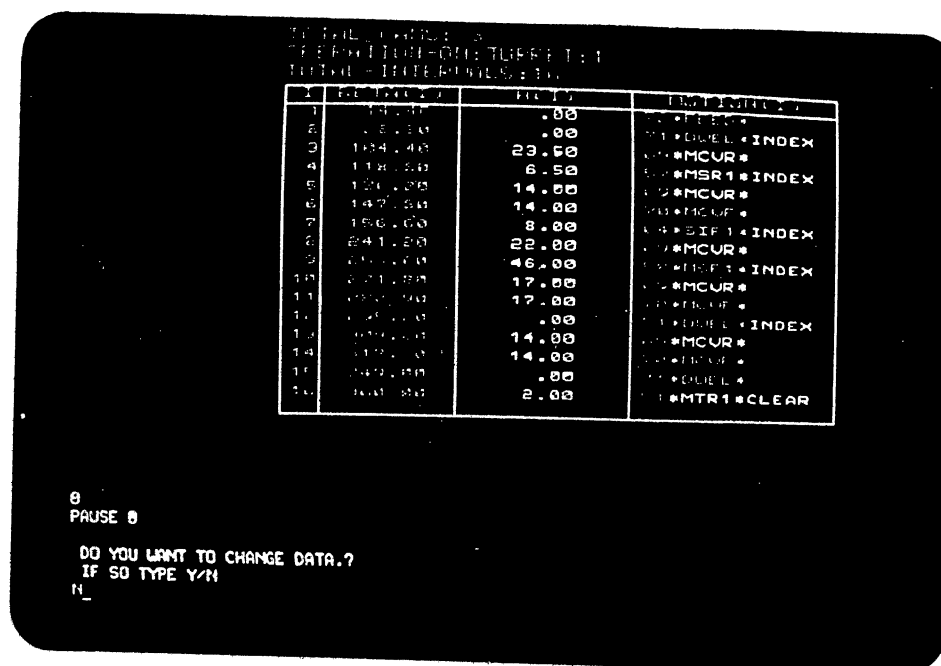


Fig. 3.6 Photograph Showing the Display of a Sample Work Dependent Input Data

98347

display, y-scale magnification can be obtained by using control zooming operation. In all the graphical displays meaningful colour matching and combinations are done to avoid confusions. The following chapter may give a still better understanding about the nature of the outputs displayed by the package.

CHAPTER 4

RESULTS AND DISCUSSIONS

The presently developed package was tested for a number of examples and the results obtained are found to be flawless. This can be attributed mainly to the screw theoretic and differential geometry approaches used for the evaluation of profile coordinates and radii of curvature respectively. The radii of curvature results when verified for dwell, are found to be exact upto the fourth decimal place. However it should be noted that during the idle strokes of indexing and clearing off operations, it is assumed that the cam follower arm will be lifted by some other mechanism and in the process the cam rotates.

In the previous chapter, a brief description about the feeding of the data and some illustrative photographs of the terminal displays were discussed. In this section, mainly the results pertaining to the typical workpieces found in the references [9] and [4] have been illustrated and analysed. The Appendix I is nothing but the cam layout sheet of the considered example taken from the reference [9]. Also it is for this particular example, the illustrations, pertaining to inputting of the data interactively, has been discussed in the previous chapter. Only for this example a number of terminal displays has been furnished. For the second example, taken from the reference [4], the important displays pertaining to turret cam design has been presented.

Taking the first example, it is very clear from the cam layout sheet three cams have been used for producing this workpiece. Considering first, the lead (turret) cam, the values of the machine dependent parameters fed are as per the Figure 3.2. During feeding of the data, the package provided all possible interactions indicating the status at every interval. Figure 3.5 shows a part of one such interaction. The display of the final work dependent input table is given in the Figure 3.6. Then the package takes care of restrictions imposed on pressure angle and radius of curvature optionally. At this stage the package indicates the status of these two important parameters. In case, the values go out of the way, the package provides in a table the exceeding values, their zone of occurrences etc., with the clear suggestions, regarding the kind of measures to be employed in order to contain them within the constraints imposed. One such event during designing of the considered example is shown in the Figure 4.1.

After redesigning, the display of the cam as per conventions followed and is shown in the Photograph 4.1. The package in this display does the following essential functions for better visualisation of an automat cam.

- graduates maximum profile radius circle from the starting point for hundred divisions.
- shows the operation zones through angular partitions and numbers them.

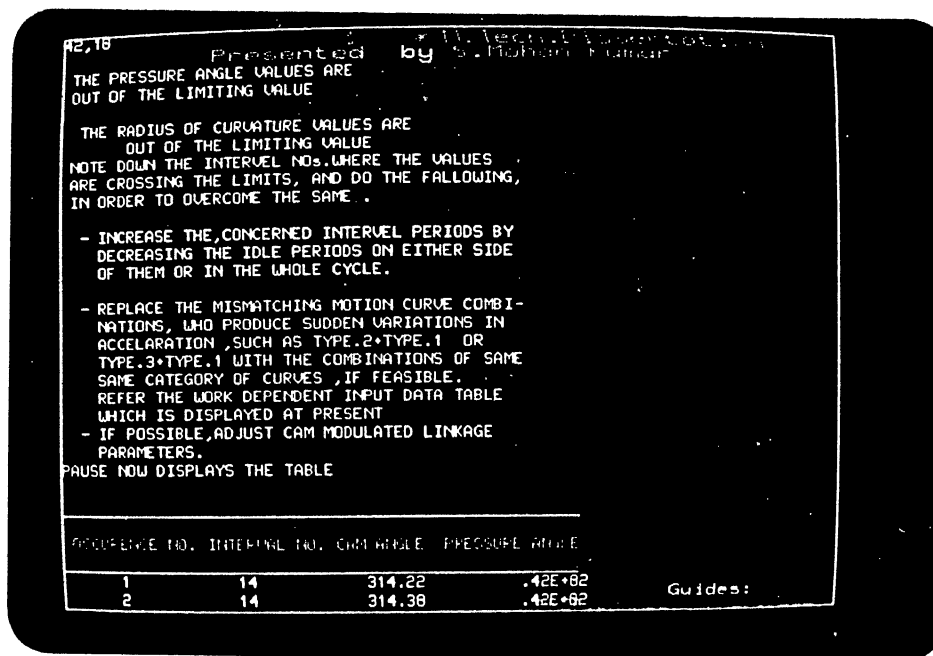


Fig. 4.1 Photograph Showing the Display of Interactions While Imposing Restrictions on Ψ and ρ

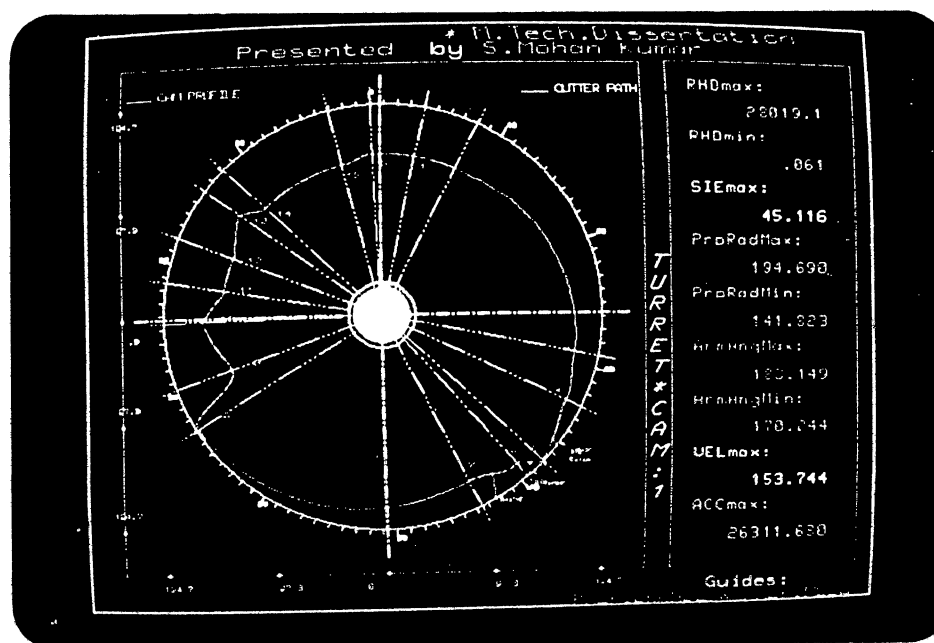


Fig. 4.2 Photograph Showing the Display of Turret Cam Profile

- mounts and graduates the scales in X and Y directions for the maximum profile radius circle.
- displays the maximum or(and) minimum values of all the important parameters.
- provides the optional display of the pressure angle, normal vector and force vector at any point on the cam surface. The values of the pressure angle at these optional inputted cam points are shown in the dialogue area.
- and provides the optional display of the cutter path for any input cutter radius (within the limitations of graphic area).

Figure 4.2 depicts the cam display for the considered example, without the cutter path. Figure 4.3 gives the magnified (zoomed) view of the normal vector, force vector and pressure angle at some selectively fed points. Figure 4.4 shows the cutter path for the optionally selected cutter radius.

Next display is about the inputted motion curves of displacement, velocity and acceleration with optional indication of the magnitudes of their scales. The naming of the idle operations is also provided in this display.

Figure 4.5 shows the photograph of the display obtained for the considered example. Similarly Figure 4.6 shows the plots of pressure angle, radius of curvature and profile radius of the discussed example. In this display, optionally one can know about the variation of contact point angle on the

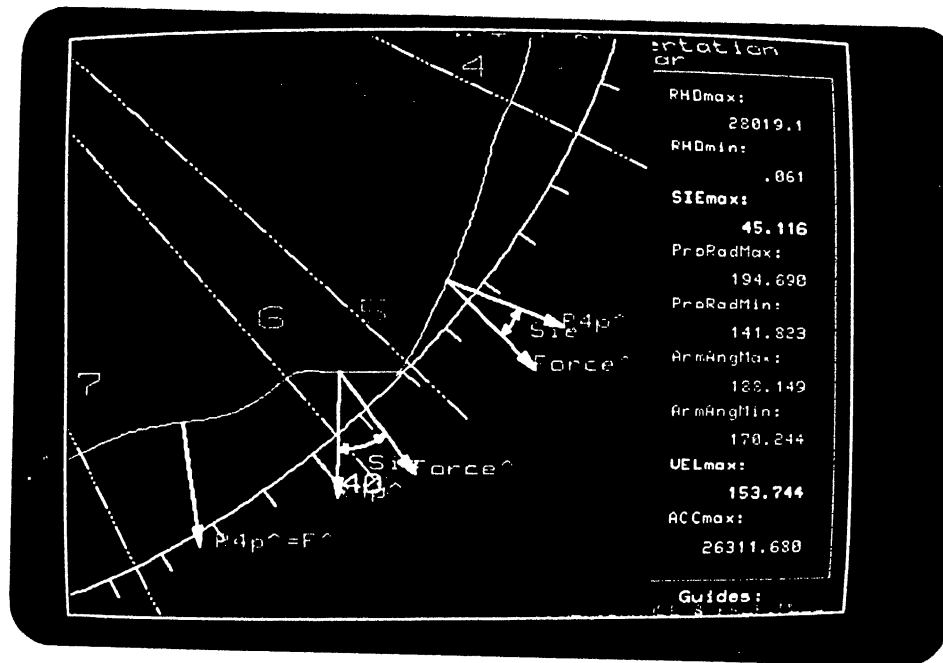


Fig. 4.3 Photograph Showing the Zoomed View of Normal Vector, Pressure Angle and Force Vector

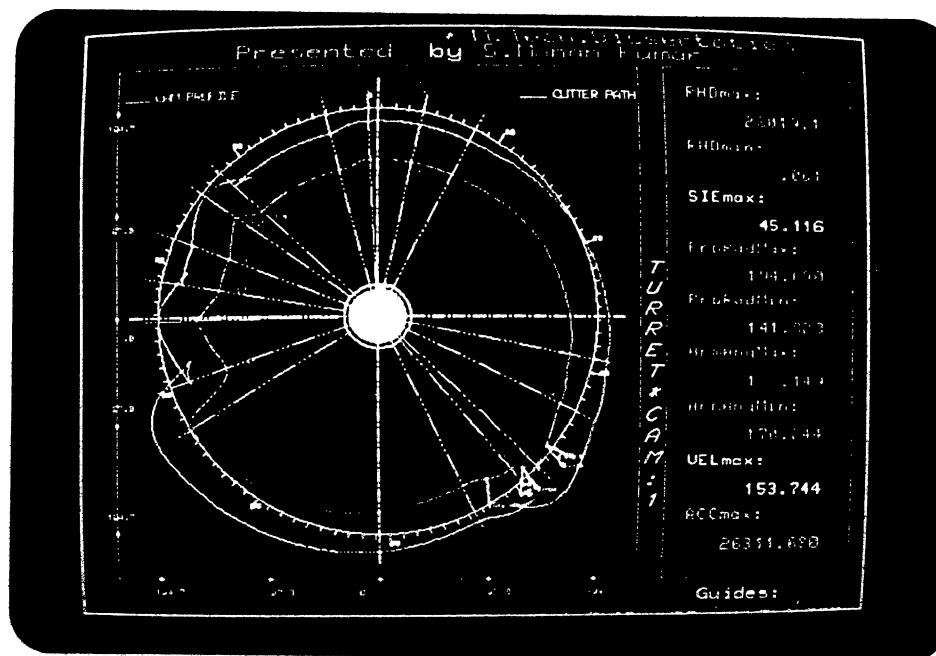


Fig. 4.4 Photograph Showing the Display of Turret Cam with Cutter Path

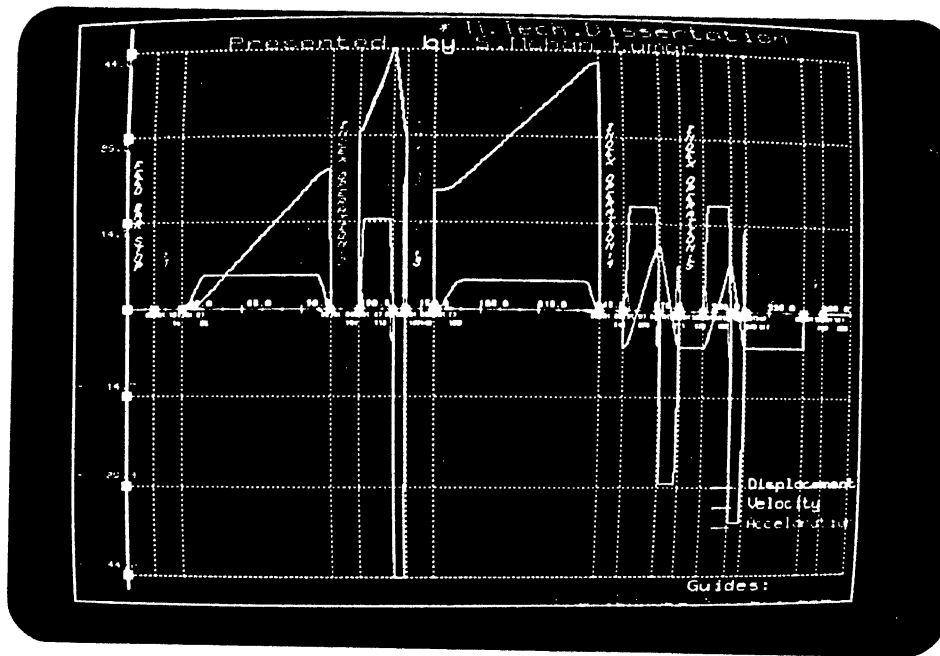


Fig. 4.5 Photograph Showing the Display of Input Motion Curve Plots S , V , A vs. θ_4 for the Turret Cam Design

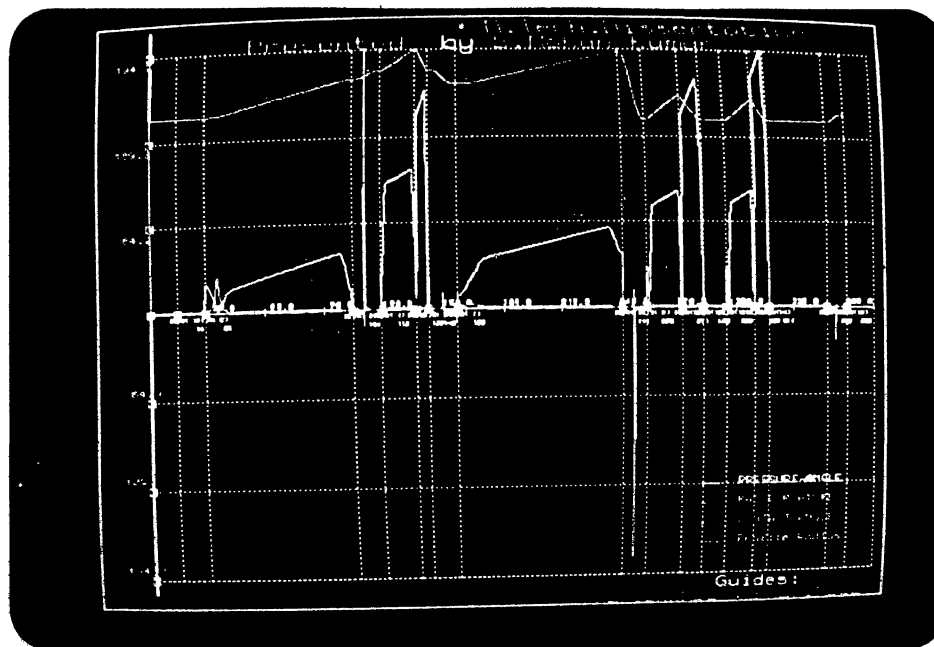


Fig. 4.6 Photograph Showing the Display of the Plots of Ψ , ρ , R vs. θ_4 for the Turret Cam Design

surface of the follower roller by getting the plot of contact angle. In the radius of curvature plot, because of its huge magnitude most of the variations appear to occur at the reference X-axis. To get a clear idea of the same, control zooming has been performed, to get the magnification of the same along Y-axis as illustrated in the Figure 4.7.

With the above displays, the outputs for the turret cam of the considered example is over. After this the user can get back all the displays outputted so far, any number of times, by simply pressing the return key. After this section, all the values of important parameters like cutter coordinates, radius of curvature, pressure angle, profile radius and cutter coordinates are started in a tabular form against the cam rotation angle.

After this the cycle of designing of the next cam starts.

The next cam chosen for designing in this example is for the front cross slide. The terminal outputs obtained in this case are as follows:

- Figure 3.3 shows the inputted machine dependent parameters.
- Figure 4.8 depicts the work dependent input data.
- Figure 4.9 shows the cam profile.
- Figure 4.10 contains S , V , A motion curve plots.
- and Figure 4.11 shows the corresponding plots of γ , ρ and R for the considered example.

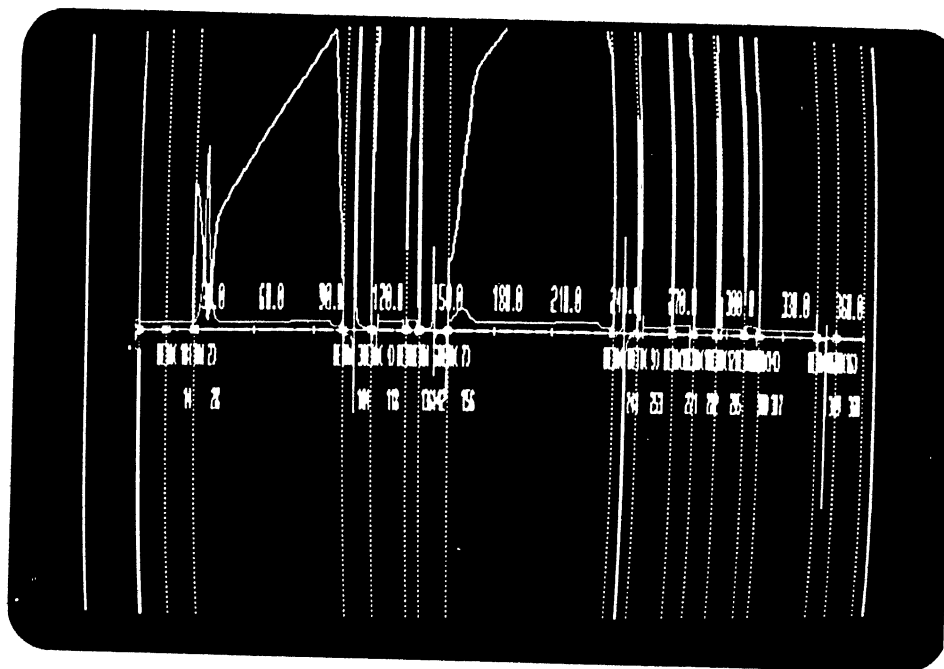


Fig. 4.7 Photograph Showing the Zoomed View of Radius of Curvature Variation

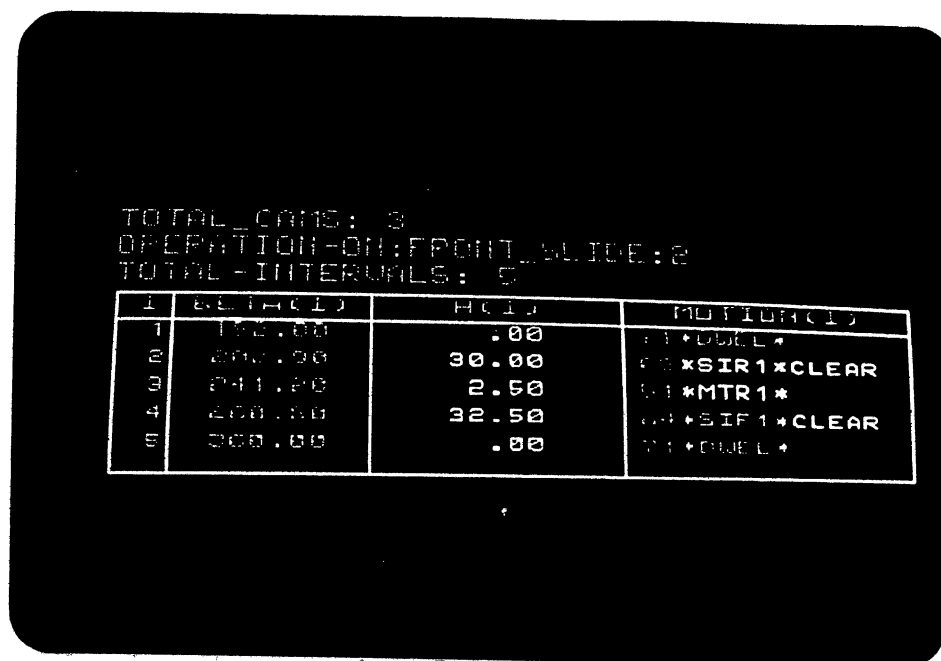


Fig. 4.8 Photograph Showing the Display of Work Dependent

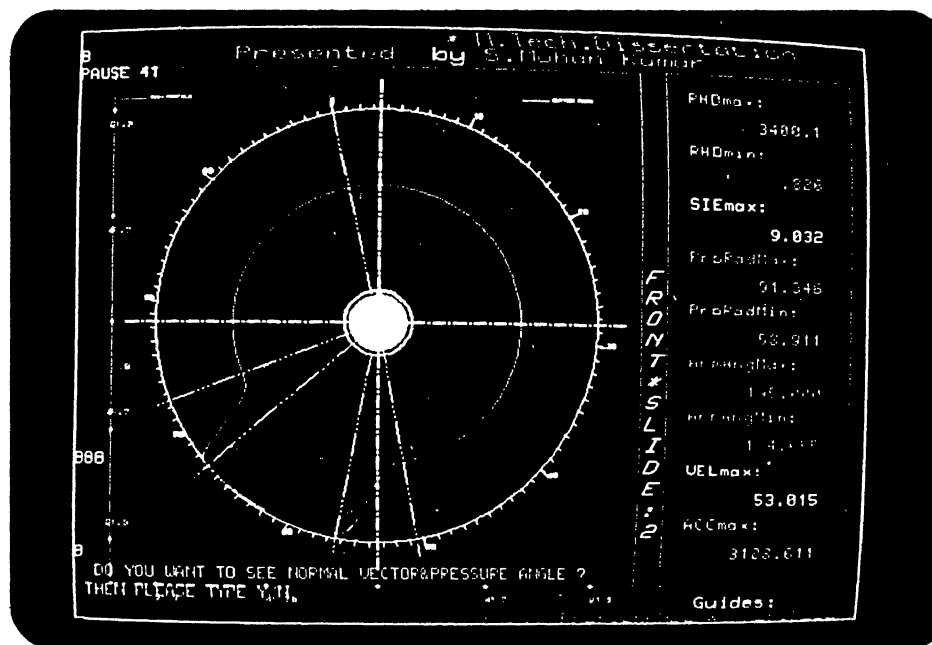


Fig. 4.9 Photograph Showing the Display of Front Slide Cam Profile

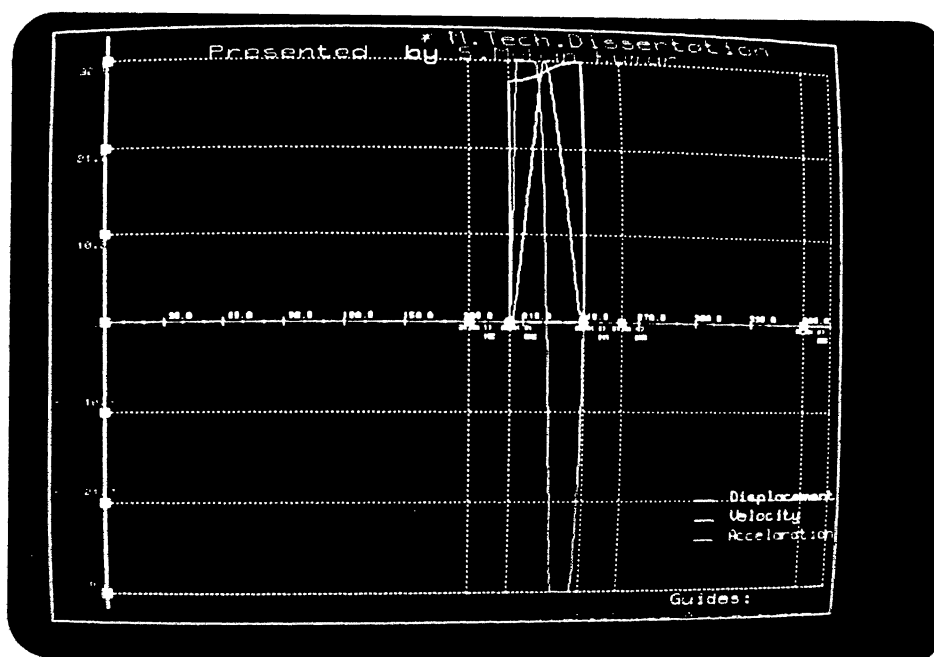


Fig. 4.10 Photograph Showing the Input Motion Curve Plots S, V, A vs. θ_4 for the Front Slide Cam Design

Finally, the process of interactive designing of the rear slide cam follows. For this cam only the displays pertaining to machine dependent input data, the work dependent input data and the cam are provided. Figures 3.4, 4.12 and 4.13 show the above displays in order.

The next example is about a typical workpiece produced on an automatic screw machine of Brown and Sharpe Company [4]. The displays of the machine dependent input, the work dependent input, the cam and the output parameter plots (of ψ , ρ and θ_2) obtained for the design of turret motion are only presented in this section. Figures 4.14, 4.15, 4.16 and 4.17 correspond to the above displays.

With the illustration of the above displays for the considered examples, it is very clear that any cam designed using this package for single spindle automatic machines in general will be complete in all respects.

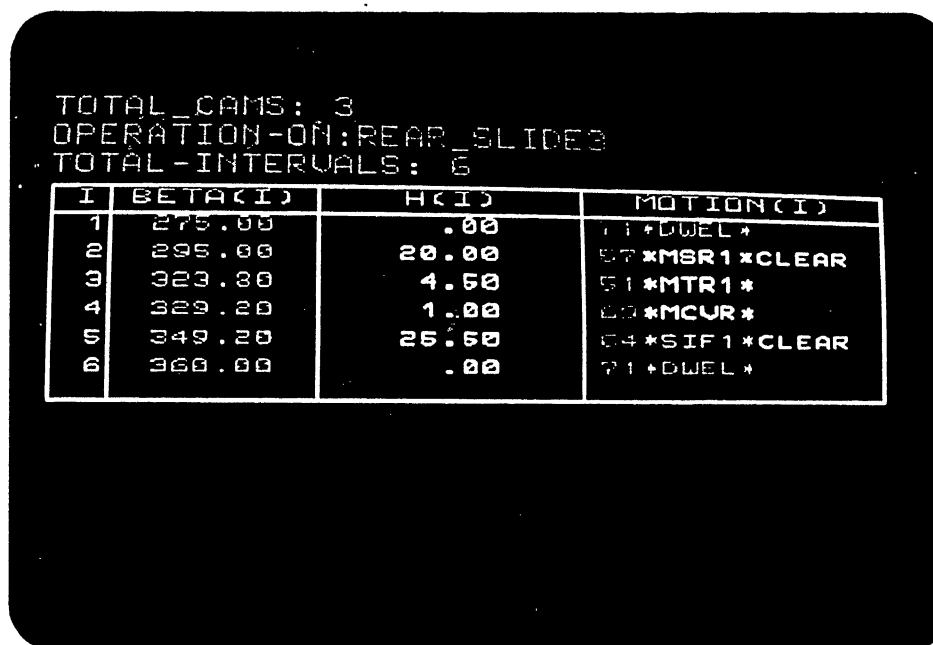


Fig. 4.12 Photograph Showing the Display of Work Dependent Input Data of Rear Slide Cam Design

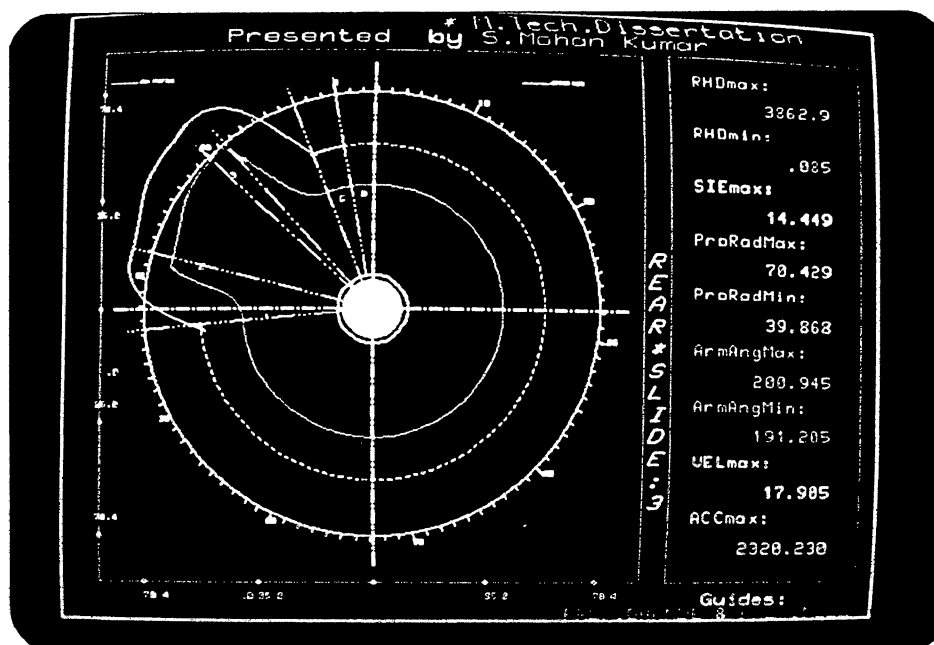


Fig. 4.13 Photograph Showing the Display of Rear Slide Cam Profile

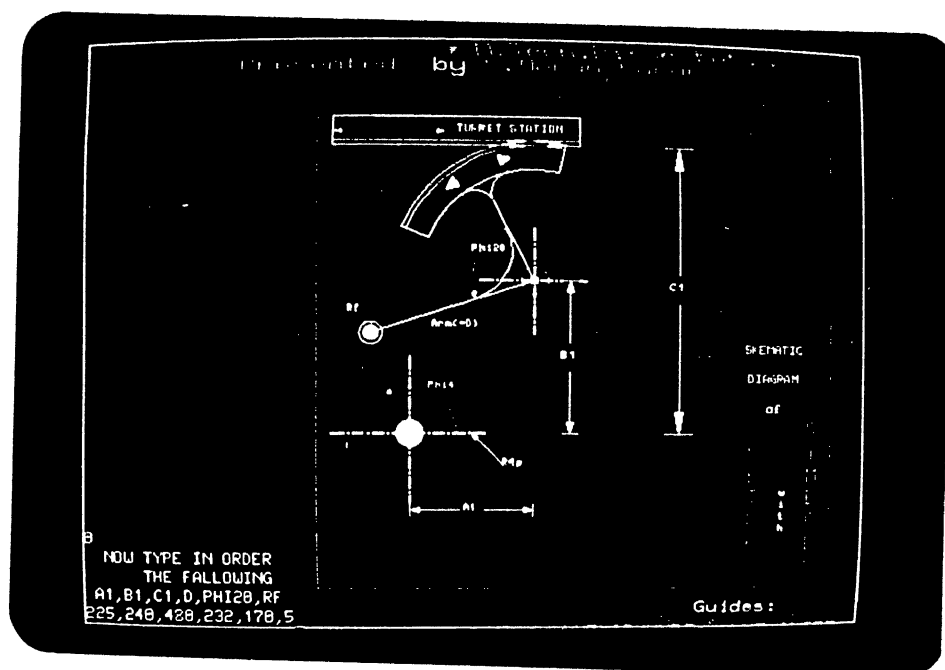


Fig. 4.14 Photograph Showing the Display of Machine Dependent Data for the Turret Cam Design of Example 2

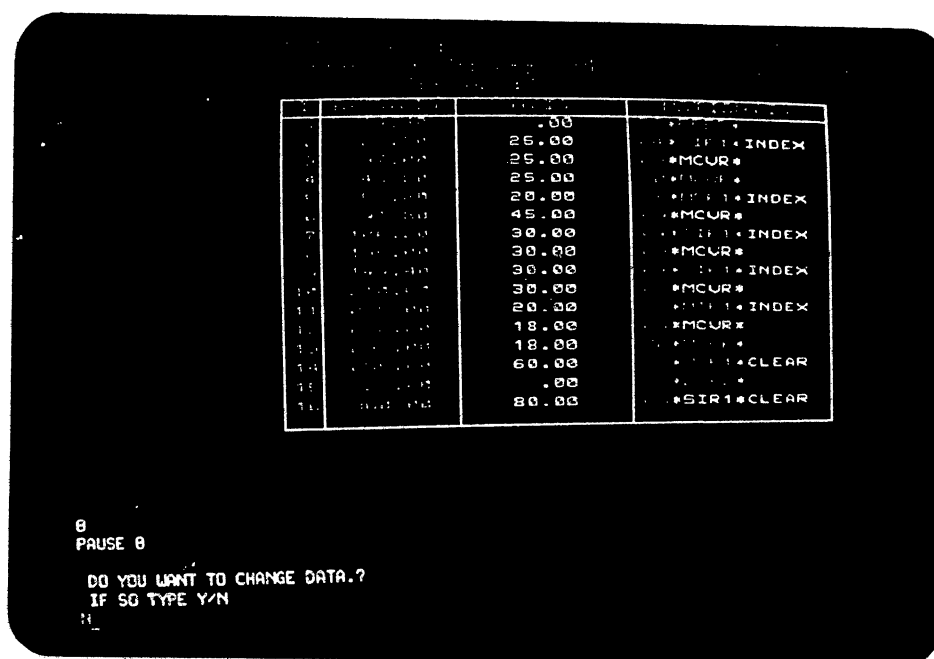


Fig. 4.15 Photograph Showing the Display of Work Dependent Input Data for Turret Cam Design of Example 2

CHAPTER 5

SUMMARY

5.1 Conclusions

An interactive package for designing work dependent cams for single spindle automats has been developed and analysed.

The package takes the essential inputs in the form of work independent data which consists of cam modulated linkage parameters and the work dependent data from the cam layout sheet. The operation dependent motion curves used in the package are developed specifically for designing cams for automats as they are being accepted universally and are found to be more useful than any other trigonometric curves or polynomial curves in the relevant field. The idea of using the normalised values of motion curves has made the computation and comparison times to a bare minimum.

The results thus obtained are highly accurate from the manufacturing aspects also. It is found that by iterative designing procedure adopted in the package, the plate cams for any complicated workpieces could be obtained very quickly and accurately to the requirements of the cam layout sheet.

The presently developed package gives the cam display strictly as per the conventions followed in the industries and

gives detailed informations about all the important parameters including coordinates.

In overall, the presently developed method may find a place in the present day industries wherein the frequent change in workpiece design are experienced.

5.2 Scope for Further Work

The subject of designing and manufacture of cams for automatic machines essentially involve three basic steps as follows:

- (1) If the detailed drawing of a component to be manufactured in an Automatic machine is given, then the first and foremost step is to output a cam layout sheet (i.e., combined tool layout and operation work sheet). This task requires a two way communication of graphical information. Also this stage should contain all possible general empirical relations to decide about the necessary cam throws, cam interval angles, cutting speeds and feeds, the tools to be used, the change of gears to be employed, minimum and maximum profile radii of the cam to be used, the machine dependent data to be decided etc.
- (2) The next stage, after possessing a cam layout sheet, is to design the cams interactively and to fulfil, all the manufacturing requirements.
- (3) The final stage, though not an essential one, is becoming a necessity for the present day industries.

After executing, the second step stated above, software for automatic production of punched tapes are to be devised for the control of the numerically controlled milling machines used for cam production.

Out of the above three topics, the present work finds its place in the second one. However there is lot of scope for further research in all these three topics, as the present work deals about the cam design for single spindle automatic machines only. There is a greater need for computer aided design and manufacturing of three dimensional, as well as two-dimensional cams, which may be work dependent or work independent for automatic machines in general, which include single and multi-spindle category.

REFERENCES

1. J. Chakraborty and S.G. Dhande, "Kinestatics and Geometry of Planar and Spatial Cam Mechanisms", Wiley Eastern Limited, 1977.
2. C.N. Neklutin, "Mechanisms and Cams for Automatic Machines", American Elsevier Publishing Company, INC, New York, 1969.
3. Fan Y. Chen, "Mechanics and Design of Cam Mechanisms", Pergamon Press, INC, 1982.
4. Brown and Sharpe Mfg. Co., "Construction and Use of Brown and Sharpe Automatic Screw Machines", North Kingstown, Rhode Island, U.S.A., 1965.
5. B.L. Boguslavsky, "Automatic and Semi-Automatic Lathes", Peace Publishers, Moscow.
6. N. Ignatyev, A. Kakoilo, V. Khomyakov, Yu. Mikheyev, "Machine Tool Design: Vol. 4", MIR Publishers, Moscow, 1969.
7. W.A.J. Chapman, "Workshop Technology: Part III", ELBS and Arnold Publishers Ltd., 1961.
8. N. Chernov, "Machine Tools", MIR Publishers, Moscow, 1975.
9. Manfred Weck, "Handbook of Machine Tools: Vol. 3: Automation and Control", Wiley Heyden Ltd., 1984.
10. Ram Kumar, "Programming Fortran 77", Data McGraw-Hill Publishing Company Ltd., 1986.
11. S.E. Goodman and S.T. Hedetniemi, "Introduction to the Design and Analysis of Algorithms", McGraw-Hill, INC, 1977.
12. V. Rajaraman, "Computer Programming in FORTRAN IV", Prentice-Hall of India Private Ltd., New Delhi, 1982.
13. Daniel D. McCracken, "Computing for Engineers and Scientists with Fortran 77", John Wiley & Sons, INC, 1984.
14. Steven Harrington, "Computer Graphics", McGraw-Hill Book Company, Singapore, 1983.
15. David F. Rogers, "Procedural Elements for Computer Graphics", McGraw-Hill Book Company, Singapore, 1985.

AN EXAMPLE OF TYPICAL CAM LAYOUT SHEET

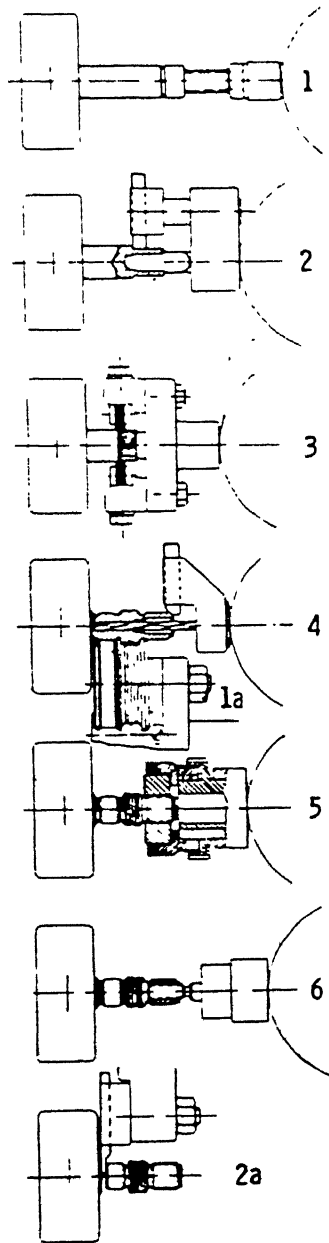
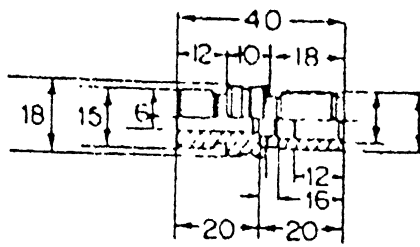


FIG. 1 TOOL LAYOUT

71

Component drawing				Material brass rod 18 DIN 1756 Ms 58							
				Spindle rev min	Turning		3000				
					External threading		1500				
					Internal threading		1500				
				Cutting speed, m min	Turning		170				
					External threading		70				
					Internal threading		47				
Number of revolutions required for one component						746					
Cycle time in s						15					
Operations				Travel	Feed per revolution of work spindle	Revolutions of work spindle		Hundredth of cam- profile			
						For the operation	To be allowed for	For idle operations	For operations	From	To
Turret	1	Feed material to stop					4		0	4	
		Turret indexing					4		4	8	
	2	Drilling and turning and threading	23.5	0.15	157	157		21	8	29	
		Turret indexing					4		29	33	
	3	Knurling	forward	12	0.4	30	30		4	33	37
			return	12	0.6	20	20		2.5	37	39.5
		Turret indexing					4		39.5	43.5	
	4	Through drilling and chamfering	22	0.125	175	175		23.5	43.5	67	
		Turret indexing					4		67	71	
	5	External thread	Cutting	17 thr		34	34		4.5	71	75.5
		Run off	17 thr		17	17		2.5	75.5	78	
		Turret indexing					4		78	82	
6	Internal thread	Tapping	14 thr		28	28		3.5	82	85.5	
		Run off	14 thr		14	14		2	85.5	87.5	
Cross slides	1a	Front cross-slide	Form cutting	2.5	0.025	100	(100)		(13)	54	67
				4.5	0.075	60	60		8	87.5	96.6
	2a	Rear cross-slide	Part-off	1	0.1	10	10		1.5	95.5	97
				Spare revolutions after part-off						3	
								545	27	73	
Revolutions per component				$\frac{545 \times 100}{73} = 746$							
Cycle time				$\frac{60 \times 746}{3000} = 15 \text{ s}$							

APPENDIX II

EQUATIONS OF MOTION CURVES

This appendix contains the motion curve equations developed for rise of all categories used in the package. For obtaining the fall curves simple mirror image transformations will suffice (Figures IIs, IIb, IIc and IId).

For all the motion curve plots the following general notations have been assumed:

Θ : the incremental angle

β : the total interval angle

γ : the ratio Θ/β

and h : the stroke length.

The equations of motion curves for:

(a) Modified Constant Velocity Rise (MCVR) are:

For the interval $0 \leq \gamma \leq (\Theta/\beta_1)$

$$S = h \left[\gamma - \frac{1}{\pi} \sin(\pi \gamma) \right] \cdot h_1$$

$$V = \frac{h}{\beta} \left[\frac{h_1}{\beta_1} (1 - \cos(\pi \gamma)) \right]$$

$$A = \frac{h}{2} \left[\left(\frac{h_1}{\beta_1^2} \right) (\pi \sin \pi \gamma) \right]$$

For the interval $(\Theta/\beta_1) \leq \gamma \leq (\Theta/(\beta_1 + \beta_2))$

$$S = h \left[h_1 + 2 \cdot (h_1/\beta_1) (\gamma - (\beta_1/\beta)) \right]$$

$$V = h/\beta \left[2 \cdot h_1/\beta_1 \right]$$

$$A = 0$$

For the interval $(\theta/(\beta_1 + \beta_2)) \leq \gamma \leq 1$

$$S = h \left[h_1 + h_2 + h_3 \left(\gamma + \frac{1}{\pi} \sin(\pi\gamma) \right) \right]$$

$$V = \frac{h}{\beta} \left[h_3 / \beta_3 (1 + \cos(\pi\gamma)) \right]$$

$$A = -\frac{h}{\beta^2} \left[h_3 / \frac{2}{3} (\pi \cdot \sin(\pi\gamma)) \right]$$

where, the values of h_1 , h_2 , h_3 and β_1 , β_2 , β_3 are determined by applying the boundary conditions at the points 0, 1, 2 and 3. Their values for unity rise (i.e., $h = 1$) are as follows:

$$\begin{array}{ll} h_1 = 0.1 & \beta_1 = 0.1739130 \\ h_2 = 0.85 & \text{and } \beta_2 = 0.7391304 \\ h_3 = 0.05 & \beta_3 = 0.0869565 \end{array}$$

(b) Modified Trapezoidal Rise Type I (MTR 1) are:

For the interval $0 \leq \gamma \leq 1/8$

$$S = 0.09724612 * h \left[4\gamma - \frac{1}{\pi} \sin(4\pi\gamma) \right]$$

$$V = 0.3889845 * h/\beta \left[1 - \cos(4\pi\gamma) \right]$$

$$A = 4.888124 * h/\beta^2 \left[\sin(4\pi\gamma) \right]$$

For the interval $1/8 \leq \gamma \leq 3/8$

$$S = h \left[2.44406184 \gamma^2 - 0.22203097 \gamma + 0.00723407 \right]$$

$$V = h/\beta \left[4.888124 \gamma - 0.3889845 \cos(4\pi\gamma - \pi) \right]$$

$$A = h/\beta^2 \left[4.888124 \sin(4\pi\gamma - \pi) \right]$$

For the interval $3/8 \leq \gamma \leq 1/2$

$$S = h \left[1.6110154 \gamma + 0.0309544 \sin(4\pi\gamma - 2\pi) - 0.3889845 \right]$$

$$V = h/\beta \left[1.6110154 + 0.3889845 \cos(4\pi\gamma - 2\pi) \right]$$

$$A = -h/\beta^2 \left[4.888124 \sin(4\pi\gamma - 2\pi) \right]$$

For the interval, $5/8 \leq \gamma \leq 7/8$

$$S = h(4.6660917\gamma - 2.44406184\gamma^2 - 1.2292648)$$

$$V = h/\beta \left[(4.6660917 - 4.888124\gamma) \right]$$

$$A = -4.888124 h/\beta^2$$

For the interval $7/8 \leq \gamma \leq 1$

$$S = h \left[0.6110154 + 0.3889845\gamma + 0.0309544 \sin(4\pi\gamma - 3\pi) \right]$$

$$V = h/\beta \left[0.388945 + 0.3889845 \cos(4\pi\gamma - 3\pi) \right]$$

$$A = h/\beta^2 \left[-4.888124 \sin(4\pi\gamma - 3\pi) \right]$$

(e) Modified Trapezoidal Rise Type II (MTR 2) are:

For the interval $0 \leq \gamma \leq 3/8$

$$S = h(2.02395\gamma^2)$$

$$V = h/\beta (4.0479\gamma)$$

$$A = h/\beta^2 (4.0479)$$

For the interval $3/8 \leq \gamma \leq 1/2$

$$S = h \left[1.5179625\gamma - 0.0256366 \sin(4\pi\gamma - \pi) - 0.2589814 \right]$$

$$V = h/\beta \left[1.5179625\gamma - 0.32216 \cos(4\pi\gamma - \pi) \right]$$

$$A = h/\beta^2 \left[4.0479 \sin(4\pi\gamma - \pi) \right]$$

For the interval $1/2 \leq \gamma \leq 5/8$

$$S = h \left[1.5180099\gamma + 0.0256336 \sin(4\pi\gamma - 2\pi) - 0.2589814 \right]$$

$$V = h/\beta \left[1.5180009 + 0.322121 \cos(4\pi\gamma - 2\pi) \right]$$

$$A = -h/\beta^2 \left[4.0479 \sin(4\pi\gamma - 2\pi) \right]$$

For the interval $5/8 \leq \gamma \leq 1$

$$S = h \left[4.0479 \gamma - 2.02395 \gamma^2 - 1.0239483 \right]$$

$$V = h/\beta \left[4.0479(1 - \gamma) \right]$$

$$A = -h/\beta^2 (4.0479)$$

(d) Modified Trapezoidal Rise Type III (MTR 3) are:

For the interval $0 \leq \gamma \leq 1/8$

$$S = h \left[0.3727847 \gamma - 0.02966552 \sin(4\pi\gamma) \right]$$

$$V = 0.3727847 h/\beta \left[1 - \cos(4\pi\gamma) \right]$$

$$A = 4.684556 h/\beta^2 \left[\sin(4\pi\gamma) \right]$$

For the interval $1/8 \leq \gamma \leq 3/8$

$$S = h \left[2.342275 \gamma^2 - 0.2127848 \gamma + 0.0069329 \right]$$

$$V = h/\beta \left[4.684556 \gamma - 0.2127848 \right]$$

$$A = 4.684556 h/\beta^2$$

For the interval $3/8 \leq \gamma \leq 1/2$

$$S = h \left[1.5439237 \gamma - 0.0296652 \sin(4\pi\gamma - \pi) - 0.2927846 \right]$$

$$V = h/\beta \left[1.5439237 - 0.3727851 \cos(4\pi\gamma - \pi) \right]$$

$$A = h/\beta^2 \left[4.684556 \sin(4\pi\gamma - \pi) \right]$$

For the interval $1/2 \leq \gamma \leq 5/8$

$$S = h \left[1.5818573 \gamma + 0.0266466 \sin(4\pi\gamma - 2\pi) - 0.3117514 \right]$$

$$V = h/\beta \left[1.5818573 + 0.3348515 \cos(4\pi\gamma - 2\pi) \right]$$

$$A = -4.20786 h/\beta^2 \left[\sin(4\pi\gamma - 2\pi) \right]$$

For the interval $5/8 \leq \gamma \leq 1$

$$S = h \left[4.2078681 \gamma - 2.10393 \gamma^2 - 1.1069536 \right]$$

$$V = h/\beta \left[4.2078681(1 - \gamma) \right]$$

$$A = -4.2078681 h/\beta^2$$

(e) Modified Sine Rise Type I (MSR 1) are:

For the interval $0 \leq \gamma \leq 1/8$

$$S = h \left[0.43989 \gamma - 0.03506 \sin(4\pi\gamma) \right]$$

$$V = h/\beta \left[0.43989(1 - \cos(4\pi\gamma)) \right]$$

$$A = 5.52794 h/\beta^2 \sin(4\pi\gamma)$$

For the interval $1/8 \leq \gamma \leq 7/8$

$$S = h \left[0.28005 + 0.43989 \gamma - 0.315035 \cos\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

$$V = h/\beta \left[0.43989 + 1.31967 \sin\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

$$A = h/\beta^2 \left[5.52794 h/\beta^2 \cos\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

For the interval $7/8 \leq \gamma \leq 1$

$$S = h \left[0.5601 + 0.43989 \gamma - 0.035014 \sin(2\pi(2\gamma - 1)) \right]$$

$$V = h/\beta \left[0.43989 \left[1 - \cos(2\pi(2\gamma - 1)) \right] \right]$$

$$A = h/\beta^2 \left[5.52794 \sin(2\pi(2\gamma - 1)) \right]$$

(f) Modified Sine Rise Type II (MSR 2) are:

For the interval $0 < \gamma < 1/8$

$$S = 2.2385 \gamma^2 h$$

$$V = 4.477 h/\rho$$

$$A = 4.477 h/\rho^2$$

For the interval $1/8 \leq \gamma \leq 7/8$

$$S = h \left[0.2201875 + 0.559625 \gamma - 0.2551581 \cos\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

$$V = h/\rho \left[0.559625 + 1.068805 \sin\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

$$A = h/\rho^2 \left[4.477 \cos\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

For the interval $7/8 \leq \gamma \leq 1$

$$S = h \left[4.477 \gamma - 2.2385 \gamma^2 - 1.238586 \right]$$

$$V = h/\rho \left[4.477(1 - \gamma) \right]$$

$$A = -4.477 h/\rho^2$$

(g) Modified Sine Rise Type III (MSR 3) are:

For the interval $0 \leq \gamma \leq 1/8$

$$S = h \left[0.4228832 \gamma + 0.0336519 \sin(4\pi\gamma) \right]$$

$$V = h/\rho \left[0.4228832 (1 - \cos(4\pi\gamma)) \right]$$

$$A = h/\rho^2 \left[5.3141071 h/\rho^2 \sin(4\pi\gamma) \right]$$

For the interval $1/8 \leq \gamma \leq 1/2$

$$S = h \left[0.4228832 \gamma + 0.3028616 \cos\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) + 0.2692097 \right]$$

$$V = h/\rho \left[0.4228832 + 1.2686496 \sin\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

$$A = h/\rho^2 \left[5.314107 \cos\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

For the interval $1/2 \leq \gamma \leq 7/8$

$$S = h \left[0.5814271 \gamma - 0.2650181 \cos\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) + 0.1899439 \right]$$

$$V = h/\rho \left[0.5813926 + 1.1101057 \sin\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right) \right]$$

$$A = 4.65 h/\rho^2 \cos\left(\frac{4\pi\gamma}{3} - \frac{\pi}{6}\right)$$

For the interval $7/8 \leq \gamma \leq 1$

$$S = h \left[4.65 \gamma - 2.325 \gamma^2 \right]$$

$$V = h/\rho \left[4.65(1 - \gamma) \right]$$

$$A = -4.65 h/\rho^2$$

(h) Sine Rise Type I (SIR 1) are:

$$S = h \left[\gamma - (1/2\pi) \sin(2\pi\gamma) \right]$$

$$V = h/\rho \left[1 - \cos(2\pi\gamma) \right]$$

$$A = 6.2831852 h/\rho^2 \sin(2\pi\gamma)$$

(i) Sine Rise Type II (SIR 2) are:

For the interval $0 \leq \gamma \leq 1/4$

$$S = 2.2385 \gamma^2 h$$

$$V = 4.1988 h/\rho \gamma$$

$$A = 4.1988 h/\rho^2$$

For the interval $1/4 \leq \gamma \leq 3/4$

$$S = h \left[1.0497 \gamma - 0.1063568 \sin(2\pi\gamma) - 0.015712 \right]$$

$$V = h/\rho \left[1.0497 \gamma - 0.0482597 \cos(2\pi\gamma) \right]$$

$$A = 4.1988 h/\rho^2 \sin(2\pi\gamma)$$

For the interval $3/4 \leq \tau \leq 1$

$$S = h [4.1988 \tau - 2.0994 \tau^2 - 1.0902677]$$

$$V = h/\beta [4.1988(1 - \tau)]$$

$$A = -4.1988 h/\beta^2$$

(j) Sine Rise Type III (SIR 3) are:

For the interval $0 \leq \tau \leq 1/2$

$$S = h [0.9241289 \tau - 0.1534458 \sin(2\pi\tau)]$$

$$V = h/\beta [0.9241289(1 - \cos(2\pi\tau))]$$

$$A = h/\beta^2 [5.8064735 \sin(2\pi\tau)]$$

For the interval $1/2 \leq \tau \leq 3/4$

$$S = h [1.1293176 \tau - 0.1144228 \sin(2\pi\tau) - 0.1025944]$$

$$V = h/\beta [1.1293176 - 0.7189402 \cos(2\pi\tau)]$$

$$A = h/\beta^2 [4.517235 \sin(2\pi\tau)]$$

For the interval $3/4 \leq \tau \leq 1$

$$S = h [4.517235 \tau - 2.2586175 \tau^2 - 1.2585594]$$

$$V = h/\beta [4.517235(1 - \tau)]$$

$$A = -4.517235 h/\beta^2$$

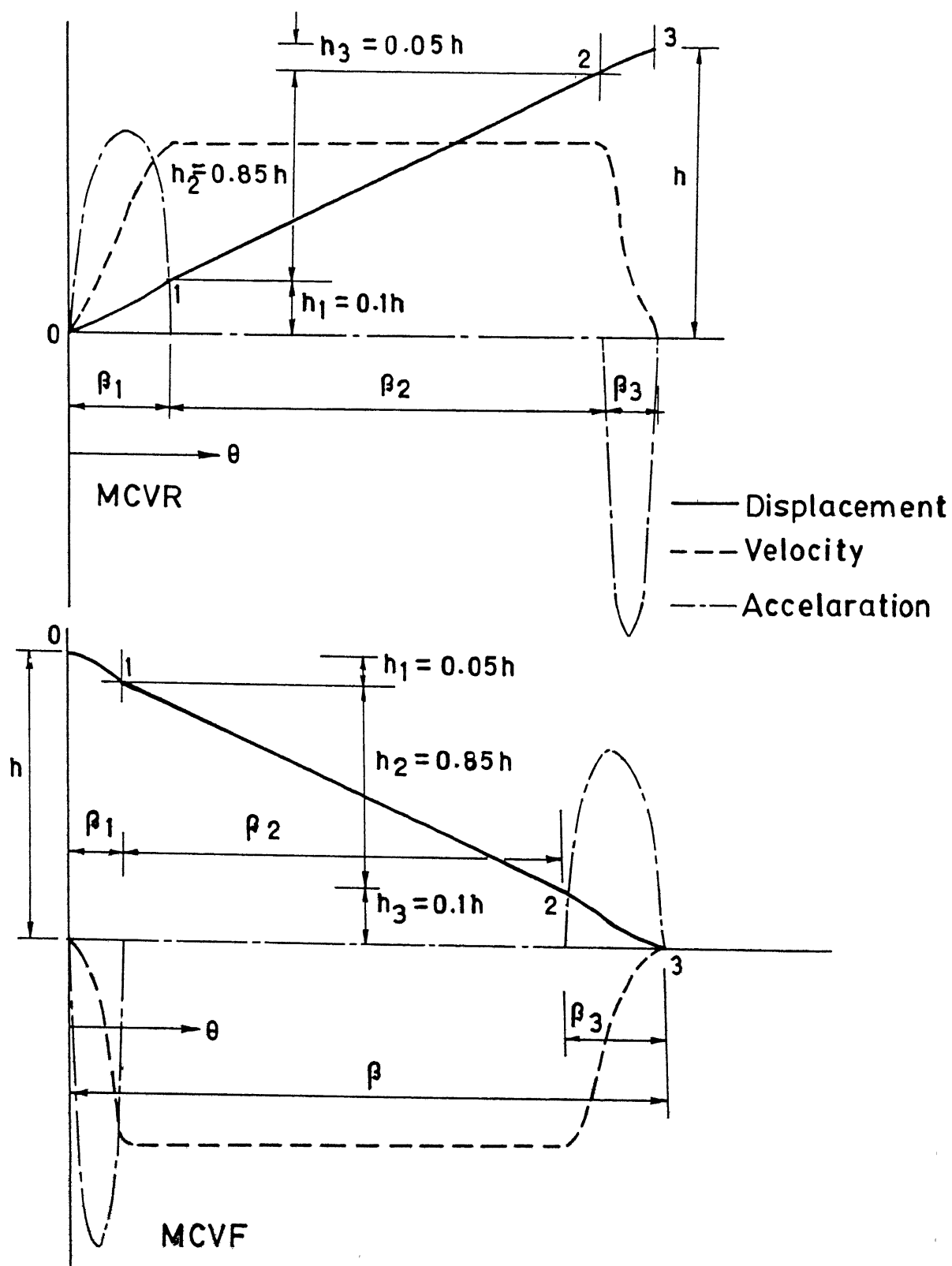


FIG. IIa MODIFIED CONSTANT VELOCITY (Rise & Fall) MOTION CURVES

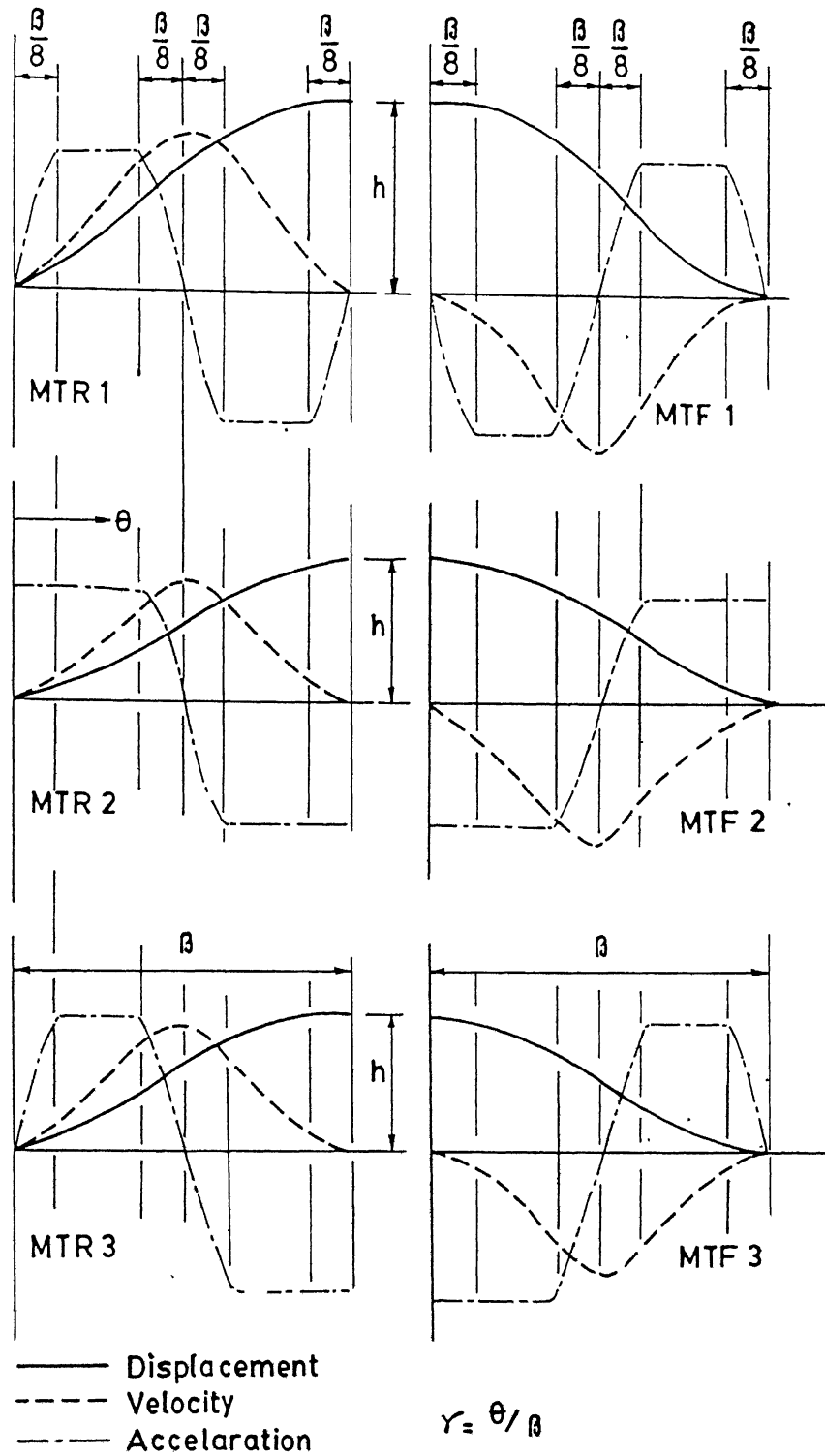


FIG. II b MODIFIED TRAPEZOIDAL (Rise & Fall) MOTION CURVES

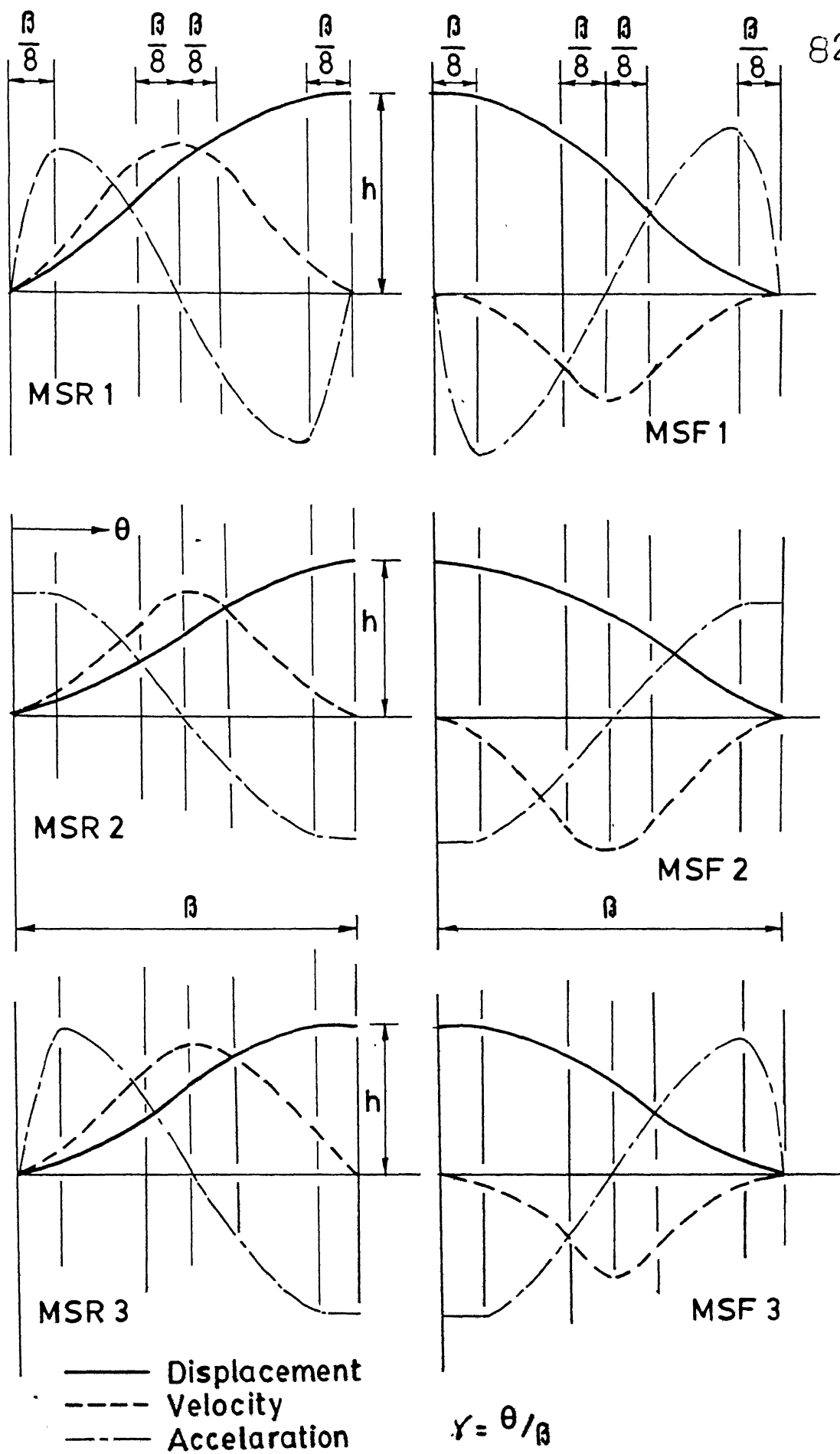


FIG. 11c MODIFIED SINE (Rise & Fall) MOTION CURVES

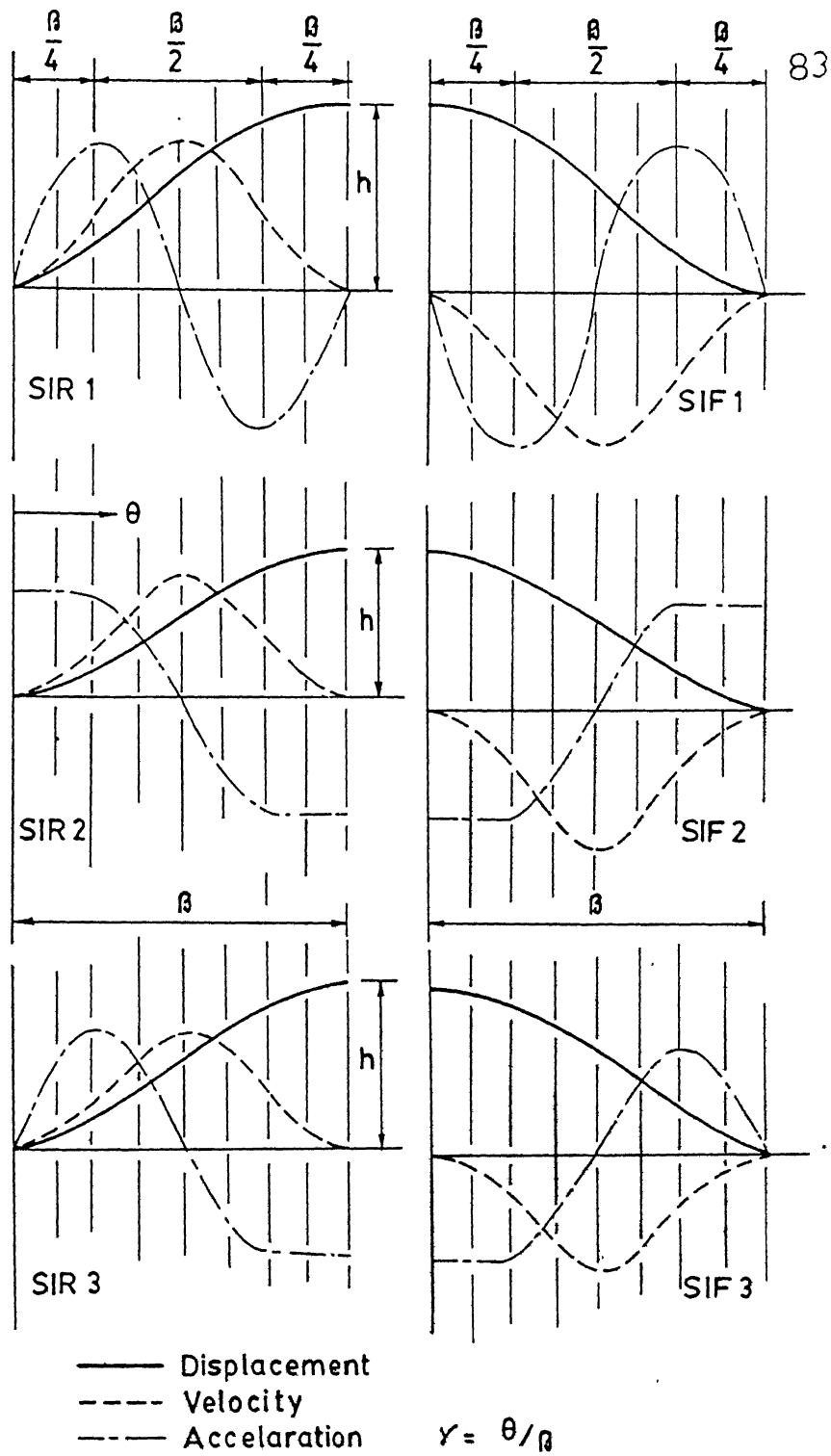


FIG. II d SINE (RISE & FALL) MOTION CURVES

APPENDIX III

CAM CURVATURE DEFINITION THROUGH DIFFERENTIAL
GEOMETRY APPROACH [1]

Consider a cam surface Σ_4 , given by $R_4^{(4)} = R_4^{(4)}(\theta, \delta)$ where θ and δ are the parameters of the surface. Then keeping δ fixed and varying θ , the θ -line on the surface are got and similarly keeping θ fixed and varying δ , the δ -lines or characteristics are obtained. The elemental vectors $R_\theta = \frac{\partial R_4^{(4)}}{\partial \theta}$ and $R_\delta = \frac{\partial R_4^{(4)}}{\partial \delta}$ are respectively tangential to the θ and δ -lines, and lie on the tangent plane at P of surface Σ_4 . Let e be a unit normal vector at the point P . Then, the normal curvature χ_n to the surface at P is given by

$$\chi_n = \frac{L d\delta^2 + 2M d\theta \cdot d\delta + N d\theta^2}{E d\delta^2 + 2F d\theta \cdot d\delta + G d\theta^2} \quad (\text{III.1})$$

The numerator of the (III.1) is known as the second quadratic form and the denominator is the first quadratic form. The coefficients of these quadratic forms are given as

$$\begin{aligned} L &= -e_\delta \cdot R_\theta \\ M &= -e_\delta \cdot R_\theta = -e_\theta \cdot R_\delta \\ N &= -e_\theta \cdot R_\theta \\ E &= R_\delta \cdot R_\delta \\ F &= R_\theta \cdot R_\delta \\ \text{and } G &= R_\theta \cdot R_\theta \end{aligned} \quad (\text{III.2})$$

where,

$$e_{\theta} = \frac{\partial e_4^{(4)}}{\partial \theta} \quad \text{and} \quad e_{\delta} = \frac{\partial e_4^{(4)}}{\partial \delta}$$

Similarly

$$R_{\theta} = \frac{\partial R_4^{(4)}}{\partial \theta} \quad \text{and} \quad R_{\delta} = \frac{\partial R_4^{(4)}}{\partial \delta}$$

Figure III shows the illustration of the symbols adopted for the discussed representations.

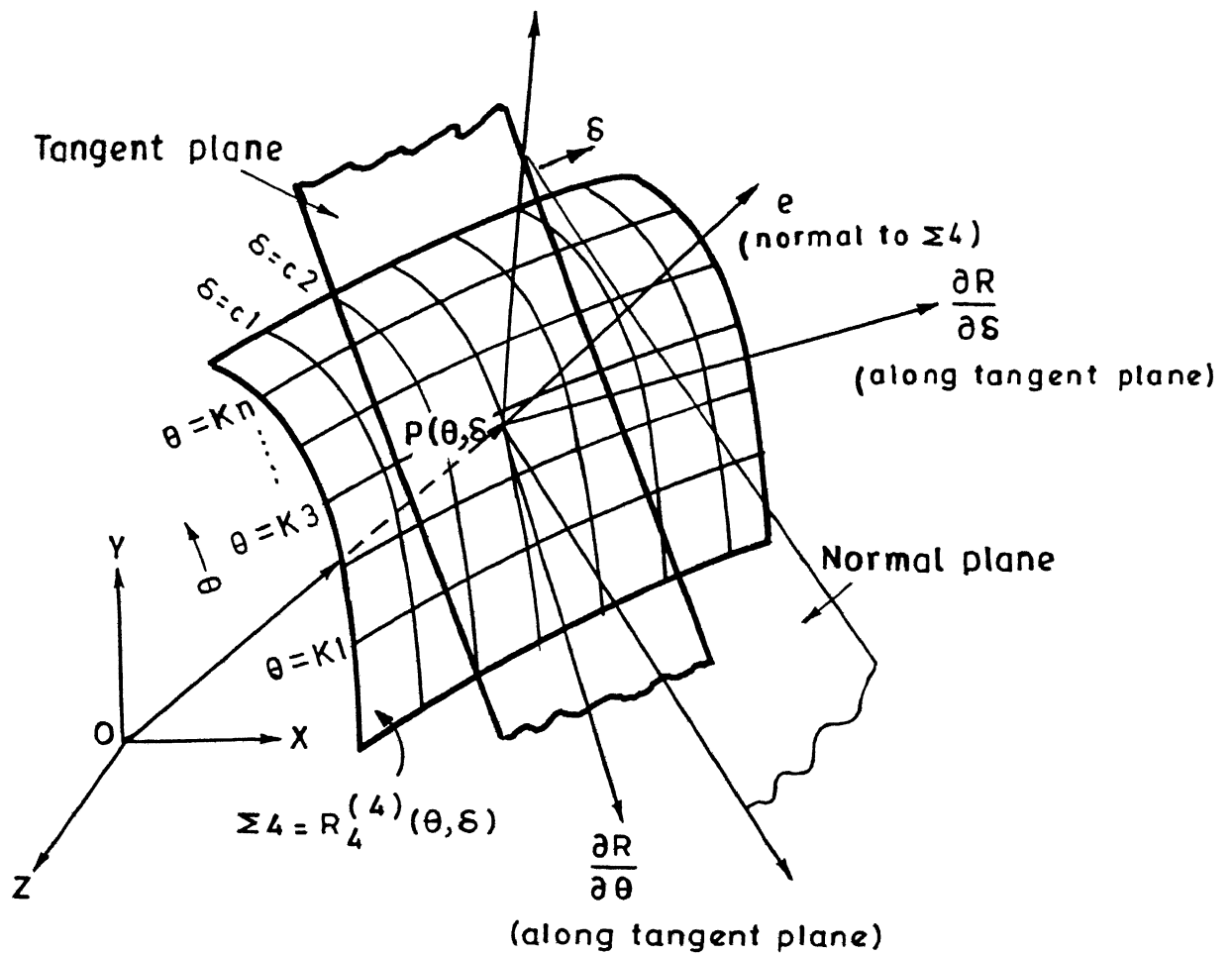


FIG. III SURFACE REPRESENTATION